Structural forecast analysis*

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Abstract

This paper shows how the structural representation of a vector autoregressive model can support forecasting. We offer a unified framework between reduced-form forecast and structural analysis, and describe how the use of the latter can help form a narrative of two reduced-form objects of real-time forecasting: the forecast errors made relative to the outturn of the data, and the revisions of the full forecast made when new data become available. We conduct a real-time exercise on the UK focusing on the inflation surge that followed the pandemic. We show that the inflation forecast was revised up not only due to contractionary supply-side shocks, but also due to a mix of expansionary demand-side shocks and a change in the underlying unconditional mean.

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1 Introduction

Vector autogressive (VAR) models are widely used both in the academic community and in policy institutions. Since their introduction by Sims (1980), a huge literature has extended and improved the VAR toolkit. This includes developing estimation procedures for inference, deriving identification techniques for causal analysis, studying the relation between VAR models and other macroeconomic models (for instance, DSGE models), and more.

One reason behind the great success of VAR models is that they provide a flexible framework for two separate types of analysis: forecasting and structural analysis. Yet, so far, the two strands of the literature on forecasting and structural modeling have largely developed separately. There is no a priori reason why VAR-based analysis should include both forecasting and structural analysis, as that depends on the research question and on the application of interest. However, the existing very limited overlap between the two strands of the literature can lead to believe that there is only limited scope for these two sides of the model to work together. One direct point of contact between the two strands of the literature is with regard to conditional forecasting, which can be simulated from structural rather than reduced form shocks in order to build scenarios (Baumeister and Kilian, 2014, Antolin-Diaz et al., 2021, Crump et al., 2025). Another example is the recent literature on optimal policy adjustments, which develops policy evaluation techniques that combine structural impulse responses and reduced form forecasts (Barnichon and Mesters, 2023, Caravello et al., 2024). We are not aware of additional work at the intersection of reduced form and structural VAR models.

This paper offers a unified framework that shows how the structural representation of a VAR model can support forecast analysis. Call T the latest time when a forecast was simulated. As already acknowledged in the literature, structural analysis is not needed to simulate the model's forecast made at time T, unless the forecast of interest is a conditional forecast based on structural shocks. Yet, we show that the structural representation of the model offers a narrative for forecast analysis, regardless of the type of forecast. We then move to the next period $T+1$. Here, new data becomes available and a new forecast is simulated, and it is possible to uncover the forecast error, which represents the difference between the forecast at time T and the data realization, along with the forecast revision over the remaining horizon, which is instead defined as the difference between the forecast produced at time T and the one computed at time $T+1$. We show that the structural representation of the model can explain the drivers of both forecast errors and forecast revisions even when the forecast of interest is a reduced-form unconditional forecast.

To appreciate the importance of structural narratives for forecast analysis, consider forecasting at policy institutions. Significant resources are invested not only in generating accurate forecasts for the economy, but also in forming a narrative that illustrates the possible economic channels consistent with the forecast produced. The narrative of a forecast is no less important than the forecast itself. A narrative is also required once the data for the new period becomes available, pinning down the forecast errors and the forecast revisions.[1] If the researcher only uses reduced form information, then forecast errors and forecast revisions can be just documented as statistical facts: one can only report to what extent the forecast made in the previous periods was incorrect, and how the forecast differs from the previous one. By contrast, we show how structural analysis can aid forecasting by taking the narrative one step forward. More specifically, we describe how forecast errors and revisions can be decomposed into *a*) the dynamic effects of the new shocks estimated at time $T+1$ with the new data realizations, *b*) the potential update in the estimate of the deterministic component of the model, for instance, a change in the unconditional mean, *c*) the change in the role of the shocks estimated until time T between the two forecasts, which could happen, for instance, due to a data revision or a change in the estimated parameters and *d*) if conditional forecast is of interest, the change in the simulated shocks generated to support the conditioning path over the forecast horizon.

We first illustrate the methodology using a bivariate simulation. We work with pseudo data on variables that, for convenience, we will refer to as output growth and inflation, driven by demand and supply shocks. The illustration starts from a period of high growth and high inflation, when the forecast at T predicts a slow reverse to the unconditional mean for both variables. When we move to the next period $T+1$, we describe that the realization of an extra data point leads to: *a*) a strong positive forecast error for output growth with respect to the forecast made at time T, but a zero forecast error for inflation, and *b*) a downward forecast revision for inflation with respect to the forecast produced in T, with the new forecast that suggests a temporary *undershooting* of inflation relative to the unconditional mean.

[1] Figure B-3 in the Online Appendix presents the series of forecasts for UK GDP and UK inflation over time produced by the Bank of England since 2020. As shown by the multiple shaded lines, there have been sizable forecast revisions over time for both GDP and inflation. The figure also shows that the time series of GDP is frequently subject to data revisions.

The traditional reduced form approach to forecasting would stop the analysis at merely documenting these changes between the two forecasts. However, these facts call for an economic interpretation. Structural analysis can help in this respect, and allows us to a) explain the positive forecast error on output growth as the joint response to expansionary demand and supply shocks, b) explain the zero forecast error on inflation in light of the matching effects on inflation of the two shocks, c) show that the negative forecast revisions for inflation in the new forecast are due to the delayed response of the deflationary supply shock that hit us between time T and $T+1$.

We then apply the methodology to data and describe how our framework can provide a narrative to the forecast paths and forecast revisions in applied work. We build a stylized four-variable SVAR model for the UK economy, and identify four shocks: a demand shock, a supply shock, an energy shock, and a monetary policy shock. We then perform a real-time evaluation exercise, focusing on the period following the Covid-19 pandemic characterized by elevated volatility and the inflation surge. For each quarter, we use the actual vintage of data, and we produce an unconditional forecast for each variable along with its decomposition. Our analysis suggests that the initial surge and positive revisions of the inflation forecast in 2022Q2 were accounted for not only by a combination of inflationary supply and energy shocks, but also by expansionary demand and monetary policy shocks, as well as an upward revision in the estimated unconditional mean.

The literature on VAR modeling is very large. We refer to Koop and Korobilis (2010) for key references of reduced form uses of the model, and to Kilian and Lütkepohl (2017) for a thorough discussion of the structural form of the model. Several methods have made use of conditional forecasting from structural models pioneered by Baumeister and Kilian (2014) and Antolin-Diaz et al. (2021), including Jarociński (2010), Bańbura et al. (2015), Moran et al. (2024) and Crump et al. (2025). Less work has been done to use the structural representation of the model to guide forecast analysis. Early traces of this approach can be found in Todd (1992), who nevertheless provides a purely narrative discussion. Giannone et al. (2004) shows some derivations similar to what we do in our paper, yet without explicit discussion of identification nor of the role of the revised shocks between different forecasts. Some recent literature has indeed used forecast errors to form a narrative of the candidate structural drivers of the errors, see for instance Giannone and Primiceri (2024). We take the analysis forward and provide a single, comprehensive framework that jointly studies real-time forecasting – which can be produced both unconditionally and conditionally, and structural analysis.[2]

The framework reported in this paper is not limited to VAR modeling. It can be extended, for instance, to dynamic factor models and to Dynamic Stochastic General Equilibrium model. We work with VARs for their high tractability, but view this framework as illustrative of the broader potential of deriving the connection between reduced form forecast analysis and structural representations, which can be derived more generally in state space models. The method is general enough that can be applied within both frequentist and Bayesian settings, and can be extended to the case in which only a subset of the structural shocks of interest are identified. In principle, the decomposition of the forecast revisions into its structural drivers offers a new dimension along which identifying restrictions can be introduced, in the spirit of narrative sign restrictions (Antolín-Díaz and Rubio-Ramírez, 2018, Giacomini et al., 2022). We leave this part of the analysis for future research.

The rest of the paper proceeds as follows section 2 illustrates the methodology. section 3 shows a bivariate illustration. section 4 shows an application to demand and supply shocks in real time. Conclusions follow.

2 Methodology

In this section we first summarize the general SVAR model used for the analysis. We then use it to show the decomposition of the forecast errors and forecast revisions. We initially work with the special case in which no data revision takes place between forecasts, and in which the population values of the model parameters are known. We then generalize the analysis to allow for data revision and parameter estimation.

[2] Earlier steps of our work were circulated in Brignone and Piffer (2025).

2.1 The model

The model is given by

$$\boldsymbol{y}_t = \sum_{l=1}^{p} \Pi_l \boldsymbol{y}_{t-l} + \boldsymbol{c} + \boldsymbol{u}_t, \tag{1a}$$

$$\boldsymbol{u}_t = B\boldsymbol{\epsilon}_t, \tag{1b}$$

$$\boldsymbol{u}_t \sim N(\mathbf{0}, \Sigma), \tag{1c}$$

$$\boldsymbol{\epsilon}_t \sim N(\mathbf{0}, I), \tag{1d}$$

$$\Sigma = BB', \tag{1e}$$

$$B = \chi(\Sigma)Q. \tag{1f}$$

where $\boldsymbol{y}_t$ is a $k \times 1$ vector of endogenous variables. $\boldsymbol{\epsilon}_t$ is a $k \times 1$ vector of structural shocks driving the data, and are assumed Normally distributed with covariance matrix normalized to the identity matrix. The reduced form innovations $\boldsymbol{u}_t$ are a linear function of the structural shocks via equation (1b). The reduced form covariance matrix Σ is functionally constrained to the $k \times k$ impact matrix of the shocks B via equation (1e). Equation (1f) relates Σ and B via the Cholesky decomposition of Σ (captured by function $\chi(.)$) and the $k \times k$ orthogonal matrix Q. Π_l represents the reduced form autoregressive parameters of the model at horizon l, $l = 1, .., p$, while $\boldsymbol{c}$ is a constant term. We refer to Arias et al. (2018) for a detailed discussion of alternative parametrizations of vector autoregressive models.

Typically, structural impulse responses are defined with respect to a single structural shock. They can be generated by simulating recursively from equations (1a)-(1b) after setting $\boldsymbol{\epsilon}_t = \boldsymbol{e}_j$, with $\boldsymbol{e}_j$ a $k \times 1$ vector of zeros except for entry j, which is set to 1. This procedure generates impulse responses to a single structural *scalar*-shock of size equal to its standard deviation. For the analysis of this paper, it is helpful to generalize this concept to a structural *vector*-shocks $\boldsymbol{\epsilon}$, where $\boldsymbol{\epsilon}$ can now take nonzero value in more than one entry. For simplicity, we refer to this impulse response as a *composite impulse response* associated with the generic impulse vector $\boldsymbol{\epsilon}$, and write it as $\boldsymbol{\phi}(h, \boldsymbol{\epsilon})$, with h the number of periods from when the shock occurs. Formally, iterate model (1a) backwards to rewrite the data at time t as a function of the data in periods $\big(t{-}1{-}\tau, .., t{-}p{-}\tau\big)$, with $\tau \geq 0$. This gives

$$\boldsymbol{y}_t = \sum_{l=1}^{p} \Pi_l^{(\tau)} \boldsymbol{y}_{t-l-\tau} + \Pi_0^{(\tau)} \boldsymbol{c}^{(\tau)} + \sum_{l=0}^{\tau} C_l B \boldsymbol{\epsilon}_{t-l}, \tag{2}$$

where the formulas for $(\Pi_l^{(\tau)}, \boldsymbol{c}^{(\tau)}, C_l)$ are available in Kilian and Lütkepohl (2017). Composite impulse responses for periods $h = 0, 1, .., \tau$ are given by

$$\boldsymbol{\phi}(0, \boldsymbol{\epsilon}) = B\boldsymbol{\epsilon}, \tag{3a}$$

$$\boldsymbol{\phi}(1, \boldsymbol{\epsilon}) = C_1 B\boldsymbol{\epsilon}, \tag{3b}$$

$$\boldsymbol{\phi}(2, \boldsymbol{\epsilon}) = C_2 B\boldsymbol{\epsilon}, \tag{3c}$$

$$\vdots \tag{3d}$$

$$\boldsymbol{\phi}(\tau, \boldsymbol{\epsilon}) = C_\tau B\boldsymbol{\epsilon}. \tag{3e}$$

By construction, if $\boldsymbol{\epsilon} = \boldsymbol{e}_j$, only shock j is subject to an impulse, and composite impulse responses coincide with conventional impulse responses.

2.2 Interpreting the forecast error and forecast revision

We are interested in how to use composite impulse responses to interpret forecast errors and forecast revisions. For this purpose, define $\boldsymbol{y}_{T+h}^{(T)}$ as the h-steps period ahead forecast made at time T, with $h = 1, .., H$ the forecast horizon. The $k \times H$ array of forecasts $Y^{(T)} = [\boldsymbol{y}_{T+1}^{(T)}, .., \boldsymbol{y}_{T+h}^{(T)}, .., \boldsymbol{y}_{T+H}^{(T)}]$ is made when the data $[\boldsymbol{y}_1, .., \boldsymbol{y}_T]$ is available. At time $T{+}1$, the data realization $\boldsymbol{y}_{T+1}$ becomes available, and $Y^{(T+1)} = [\boldsymbol{y}_{T+2}^{(T+1)}, .., \boldsymbol{y}_{T+h}^{(T+1)}, .., \boldsymbol{y}_{T+H}^{(T+1)}]$ is generated using data $[\boldsymbol{y}_1, .., \boldsymbol{y}_T, \boldsymbol{y}_{T+1}]$. Note that we hold the end of the forecast horizon at $T{+}H$ (rather than extending it to $T{+}H{+}1$) for simplicity.

We are interested in using composite impulse responses to interpret two related but different objects: the forecast error

$$\boldsymbol{v}_{T+1} = \boldsymbol{y}_{T+1} - \boldsymbol{y}_{T+1}^{(T)}, \tag{4}$$

and the forecast revision

$$\begin{pmatrix} \boldsymbol{y}_{T+2}^{(T+1)} - \boldsymbol{y}_{T+2}^{(T)} \\ \boldsymbol{y}_{T+3}^{(T+1)} - \boldsymbol{y}_{T+3}^{(T)} \\ \vdots \\ \boldsymbol{y}_{T+H}^{(T+1)} - \boldsymbol{y}_{T+H}^{(T)} \end{pmatrix}. \tag{5}$$

In words, the forecast error reports the difference between the data realization at time $T+1$ and the forecast made for that period at time T. The forecast revision, instead, is the change in the forecast over the rest of the forecast horizon.[3]

Define $U^{(T)} = [\boldsymbol{u}_{T+1}^{(T)}, .., \boldsymbol{u}_{T+h}^{(T)}, .., \boldsymbol{u}_{T+H}^{(T)}]$ the reduced form innovations simulated to generate the forecast $Y^{(T)}$, and define $E^{(T)} = [\boldsymbol{\epsilon}_{T+1}^{(T)}, .., \boldsymbol{\epsilon}_{T+h}^{(T)}, .., \boldsymbol{\epsilon}_{T+H}^{(T)}]$ the underlying simulated structural shocks, with $\boldsymbol{\epsilon}_{t+h}^{(T)} = B^{-1}\boldsymbol{u}_{t+h}^{(T)}$. Three cases summarize the alternative approaches to forecasting with VAR models: a) if $Y^{(T)}$ is an unconditional forecast, the researcher draws $U^{(T)}$ from the distribution (1b), sometimes directly setting $U^{(T)}$ equal to zero; b) if $Y^{(T)}$ is a conditional forecast simulated from *reduced* form shocks, the researcher draws $U^{(T)}$ from equation (1b) subject to linear restrictions that ensure the conditioning path of interest (Waggoner and Zha, 1999); c) if $Y^{(T)}$ is a conditional forecast simulated from *structural* shocks, the researcher draws $E^{(T)}$ from equation (1d) subject to linear restrictions that ensure the conditioning path of interest (Baumeister and Kilian, 2014, Bańbura et al., 2015, Antolin-Diaz et al., 2021, Chan et al., 2025 and Crump et al., 2025). Our method works irrespectively of the type of simulated forecast as long as both $U^{(T)}$ and $E^{(T)}$ are available. For simplicity, the illustrations and applications shown in this paper only use unconditional forecasts under the special setting in which all entries of $(U^{(T)}, E^{(T)})$ are zero.

2.3 A simplified setting

In this section we help set ideas by working under selected simplifying assumptions, which we remove in the next section: we assume that a) the model includes no constant and only one lag of the endogenous variables, b) the true parameter values of the model are known, and hence also the realizations of the shocks, and c) no data revision occurs between periods. Equation (1a) hence simplifies to

$$\boldsymbol{y}_t = \Pi \boldsymbol{y}_{t-1} + \boldsymbol{u}_t. \tag{6}$$

[3] Our method can be extended to forecast errors and forecast revisions at time $T+1$ relative to forecasts made in periods *earlier* than T. For simplicity, the analysis outlined in the paper only studies the case relative to the forecast at time T.

and (Π, B, Σ) are now treated as known parameters.

$Y^{(T)}$ can be computed as

$$\begin{pmatrix} \boldsymbol{y}_{T+1}^{(T)} \\ \boldsymbol{y}_{T+2}^{(T)} \\ \boldsymbol{y}_{T+3}^{(T)} \\ \vdots \\ \boldsymbol{y}_{T+H}^{(T)} \end{pmatrix} = \begin{pmatrix} \Pi \\ \Pi^2 \\ \Pi^3 \\ \vdots \\ \Pi^H \end{pmatrix} \boldsymbol{y}_T + \begin{pmatrix} I & 0 & 0 & \dots & 0 \\ \Pi & I & 0 & \dots & 0 \\ \Pi^2 & \Pi & I & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \Pi^{H-1} & \Pi^{H-2} & \Pi^{H-3} & \dots & I \end{pmatrix} \begin{pmatrix} \boldsymbol{u}_{T+1}^{(T)} \\ \boldsymbol{u}_{T+2}^{(T)} \\ \boldsymbol{u}_{T+3}^{(T)} \\ \vdots \\ \boldsymbol{u}_{T+H}^{(T)} \end{pmatrix}, \tag{7}$$

$$= \begin{pmatrix} \Pi \\ \Pi^2 \\ \Pi^3 \\ \vdots \\ \Pi^H \end{pmatrix} \boldsymbol{y}_T + \begin{pmatrix} I \\ \Pi \\ \Pi^2 \\ \vdots \\ \Pi^{H-1} \end{pmatrix} \boldsymbol{u}_{T+1}^{(T)} + \begin{pmatrix} 0 & 0 & \dots & 0 \\ I & 0 & \dots & 0 \\ \Pi & I & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \Pi^{H-2} & \Pi^{H-3} & \dots & I \end{pmatrix} \begin{pmatrix} \boldsymbol{u}_{T+2}^{(T)} \\ \boldsymbol{u}_{T+3}^{(T)} \\ \vdots \\ \boldsymbol{u}_{T+H}^{(T)} \end{pmatrix}. \tag{8}$$

This could be a conditional or an unconditional forecast in a way reflected by the selection of $U^{(T)}$.

It is instructive to notice that at time $T{+}1$ the data realization $\boldsymbol{y}_{T+1}$ under model (6) differs from the forecast $\boldsymbol{y}_{T+1}^{(T)}$ according to equation

$$\boldsymbol{y}_{T+1} = \Pi \boldsymbol{y}_T + \boldsymbol{u}_{T+1}, \tag{9}$$

$$= \underbrace{\Pi \boldsymbol{y}_T + \boldsymbol{u}_{T+1}^{(T)}}_{\boldsymbol{y}_{T+1}^{(T)}} + \underbrace{(\boldsymbol{u}_{T+1} - \boldsymbol{u}_{T+1}^{(T)})}_{\boldsymbol{v}_{T+1}}, \tag{10}$$

$$\boldsymbol{v}_{T+1} = \boldsymbol{u}_{T+1} - \boldsymbol{u}_{T+1}^{(T)}, \tag{11}$$

$$= B(\boldsymbol{\epsilon}_{T+1} - \boldsymbol{\epsilon}_{T+1}^{(T)}). \tag{12}$$

Put differently, the forecast error $\boldsymbol{v}_{T+1}$ (which, by definition, equals the difference between the data realization $\boldsymbol{y}_{T+1}$ and the forecast $\boldsymbol{y}_{T+1}^{(T)}$) coincides with the difference between the realization of the innovation generating the data ($\boldsymbol{u}_{T+1}$) and the values used at time T to compute the forecast ($\boldsymbol{u}_{T+1}^{(T)}$). Since the innovations $\boldsymbol{u}_t$ are ultimately driven by structural shocks $\boldsymbol{\epsilon}_t$, the forecast error is driven by the difference between the actual realizations of the structural shocks behind the data at time $T{+}1$ ($\boldsymbol{\epsilon}_{T+1}$) and the values consistent with the reduced form innovations used to generate the forecast ($\boldsymbol{\epsilon}_{T+1}^{(T)}$). If equation (6) is the true model, it is the inability to correctly predict $\boldsymbol{\epsilon}_{T+1}$ that drives the forecast error made at time $T{+}1$.

For the forecast made at time $T{+}1$ until horizon $T{+}H$, it holds that

$$
\begin{pmatrix} \boldsymbol{y}_{T+2}^{(T+1)} \\ \boldsymbol{y}_{T+3}^{(T+1)} \\ \vdots \\ \boldsymbol{y}_{T+H}^{(T+1)} \end{pmatrix} = \begin{pmatrix} \Pi^2 \\ \Pi^3 \\ \vdots \\ \Pi^H \end{pmatrix} \boldsymbol{y}_T + \begin{pmatrix} \Pi & I & 0 & \dots & 0 \\ \Pi^2 & \Pi & I & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \Pi^{H-1} & \Pi^{H-2} & \Pi^{H-3} & \dots & I \end{pmatrix} \begin{pmatrix} \boldsymbol{u}_{T+1} \\ \boldsymbol{u}_{T+2}^{(T+1)} \\ \boldsymbol{u}_{T+3}^{(T+1)} \\ \vdots \\ \boldsymbol{u}_{T+H}^{(T+1)} \end{pmatrix}, \tag{13}
$$

$$
= \begin{pmatrix} \Pi^2 \\ \Pi^3 \\ \vdots \\ \Pi^H \end{pmatrix} \boldsymbol{y}_T + \begin{pmatrix} \Pi \\ \Pi^2 \\ \vdots \\ \Pi^{H-1} \end{pmatrix} \boldsymbol{u}_{T+1} + \begin{pmatrix} I & 0 & \dots & 0 \\ \Pi & I & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \Pi^{H-2} & \Pi^{H-3} & \dots & I \end{pmatrix} \begin{pmatrix} \boldsymbol{u}_{T+2}^{(T+1)} \\ \boldsymbol{u}_{T+3}^{(T+1)} \\ \vdots \\ \boldsymbol{u}_{T+H}^{(T+1)} \end{pmatrix}. \tag{14}
$$

Note that generating the new forecast $Y^{(T+1)}$ requires simulating possibly new innovations $U^{(T+1)}$ that might well differ from $U^{(T)}$, a case that might, for instance, arise if the forecast is a conditional forecast. Note also that the first line in the above equation features $\boldsymbol{u}_{T+1}$, namely the innovations responsible for the data realization $\boldsymbol{y}_{T+1}$.

Subtracting all but the first row of (8) from equation (14) highlights the following equation pinning down the forecast revision:

$$
\begin{pmatrix} \boldsymbol{y}_{T+2}^{(T+1)} - \boldsymbol{y}_{T+2}^{(T)} \\ \boldsymbol{y}_{T+3}^{(T+1)} - \boldsymbol{y}_{T+3}^{(T)} \\ \vdots \\ \boldsymbol{y}_{T+H}^{(T+1)} - \boldsymbol{y}_{T+H}^{(T)} \end{pmatrix} = \underbrace{\begin{pmatrix} \Pi \\ \Pi^2 \\ \vdots \\ \Pi^H \end{pmatrix} \underbrace{(\boldsymbol{u}_{T+1} - \boldsymbol{u}_{T+1}^{(T)})}_{\boldsymbol{v}_{T+1} = B(\boldsymbol{\epsilon}_{T+1} - \boldsymbol{\epsilon}_{T+1}^{(T)})}}_{\boldsymbol{\gamma}_1} + \underbrace{\begin{pmatrix} I & 0 & \dots & 0 \\ \Pi & I & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \Pi^{H-2} & \Pi^{H-3} & \dots & I \end{pmatrix} \begin{pmatrix} \boldsymbol{u}_{T+2}^{(T+1)} - \boldsymbol{u}_{T+2}^{(T)} \\ \boldsymbol{u}_{T+3}^{(T+1)} - \boldsymbol{u}_{T+3}^{(T)} \\ \vdots \\ \boldsymbol{u}_{T+H}^{(T+1)} - \boldsymbol{u}_{T+H}^{(T)} \end{pmatrix}}_{\boldsymbol{\gamma}_2}. \tag{15}
$$

In words, two elements are responsible for the forecast revision within the simplified setting studied in this section. The first, $\boldsymbol{\gamma}_1$, is the composite effect associated with the forecast error $\boldsymbol{v}_{T+1}$ over the full forecast horizon. This is the composite impulse response defined in the previous section, evaluated at the difference in the structural shocks driving $\boldsymbol{v}_{T+1}$, i.e. $\boldsymbol{\epsilon}_{T+1} - \boldsymbol{\epsilon}_{T+1}^{(T)}$ (equation 12), and delayed by one period (i.e. premultiplied by Π). The second, $\boldsymbol{\gamma}_2$, is the effects associated with the difference in the shocks $U^{(T)}$ and $U^{(T+1)}$ simulated to generate the two forecasts.

A special case simplifies things further and highlights the key ideas outlined so far.

Suppose that $(Y^{(T)}, Y^{(T+1)})$ are computed as an unconditional forecast that assumes zero future shocks. This corresponds to $\boldsymbol{u}_{T+h}^{(T)} = \mathbf{0}$ for $h = 1, .., H$ and $\boldsymbol{u}_{T+h}^{(T+1)} = \mathbf{0}$ for $h = 2, .., H$. Equations (12) and (15) now simplify to

$$\begin{pmatrix} \boldsymbol{y}_{T+1}^{(T)} - \boldsymbol{y}_{T+1} \\ \boldsymbol{y}_{T+2}^{(T+1)} - \boldsymbol{y}_{T+2}^{(T)} \\ \boldsymbol{y}_{T+3}^{(T+1)} - \boldsymbol{y}_{T+3}^{(T)} \\ \vdots \\ \boldsymbol{y}_{T+H}^{(T+1)} - \boldsymbol{y}_{T+H}^{(T)} \end{pmatrix} = \begin{pmatrix} I \\ \Pi \\ \Pi^2 \\ \vdots \\ \Pi^H \end{pmatrix} B \boldsymbol{\epsilon}_{T+1}. \tag{16}$$

This equation shows that the forecast error and forecast revisions (first and remaining entries of 16) coincide with the joint effects of the structural shocks that hit the system at time $T+1$. The forecast error and revisions over the full forecast horizon are nothing more than the composite impulse response defined in equation (3) evaluated at $\boldsymbol{\epsilon}_{T+1}$.

Equation (16) helps highlight the importance of the results from this section. In general, forecast errors and forecast revisions are viewed as statistics documenting either the error made in the first period of the forecast, or the update in the full remaining forecast. Yet, policy institutions always give great importance to being able to form a narrative that helps explain the forecast errors and revisions. Equation (16) helps think of the forecast errors and revisions as the output of the structural shocks that become available at time $T+1$. Since these shocks are structural, the forecast error and revision can now be decomposed into economically meaningful stochastic events, for instance as the response to demand rather than supply shocks. Forecast errors and revisions hence become economically interpretable objects.

2.4 Extension to a more general setting

The above section works under the assumption that the data generating process is a VAR model with no constant, whose parameters are known, and where no data revision occurs between T and $T+1$. We now generalize the analysis.

Call $[\boldsymbol{y}_1^{(T)}, .., \boldsymbol{y}_{T-\tau}^{(T)}, \boldsymbol{y}_{T-\tau+1}^{(T)}, .., \boldsymbol{y}_T^{(T)}]$ and $[\boldsymbol{y}_1^{(T+1)}, .., \boldsymbol{y}_{T-\tau}^{(T+1)}, \boldsymbol{y}_{T-\tau+1}^{(T+1)}, .., \boldsymbol{y}_{T+1}^{(T+1)}]$ the datasets available to compute the forecasts at time T and $T+1$, respectively. Call $(\Pi_l^{(T)}, \boldsymbol{c}^{(T)}, B^{(T)})$ the parameter values used for the forecast at time T, and $\boldsymbol{\epsilon}_t^{(T)}$, $t = 1, .., T$ the implied estimates of the structural shocks. A similar notation holds for the forecast at time

$T+1$. Then, equation (2) can be rewritten as

$$\boldsymbol{y}_{T+h}^{(T)} = \underbrace{\sum_{l=0}^{p-1} \Pi_l^{(\tau,h,T)} \boldsymbol{y}_{T-\tau-l}^{(T)}}_{\boldsymbol{dc1}_{T+h}^{(\tau,T,h)}} + \underbrace{\Pi_0^{(\tau,h,T)} \boldsymbol{c}}_{\boldsymbol{dc2}_{T+h}^{(\tau,T,h)}} + \underbrace{\sum_{l=0}^{\tau-1} C_l^{(h,T)} B^{(T)} \boldsymbol{\epsilon}_{T-l}^{(T)}}_{\boldsymbol{sc1}_{T+h}^{(\tau,T,h)}} + \underbrace{\sum_{l=1}^{h} C_l^{(h,T)} B^{(T)} \boldsymbol{\epsilon}_{T+l}^{(T)}}_{\boldsymbol{sc2}_{T+h}^{(\tau,T,h)}}. \tag{17}$$

This decomposition highlights that the forecast can be thought of as composed of four distinct parts:

- $\boldsymbol{dc1}_{T+h}^{(\tau,T,h)}$ captures the role attributed to the deterministic component from the initial condition of the data up to time $T-\tau$. Model stationarity implies that this part converges to zero as τ increases;
- $\boldsymbol{dc2}_{T+h}^{(\tau,T,h)}$ captures the role attributed to the deterministic component from the constant term up to time $T-\tau$. Model stationarity implies that this part converges to the unconditional mean of the model;
- $\boldsymbol{sc1}_{T+h}^{(\tau,T,h)}$ captures the role played by the structural shocks that were estimated between time $T-\tau+1$ and T;
- $\boldsymbol{sc2}_{T+h}^{(\tau,T,h)}$ captures the role played by the structural shocks consistent with the simulated innovations that enforce the forecast between period $T+1$ and period $T+H$.

For example, the historical decompositions in Kilian and Lütkepohl (2017) and Bergholt et al. (2024) set $h = 0$ and $\tau = T$ and interpret the stochastic component as the role associated with the structural shocks from the full sample period. Iterating equation

(17) forward to study the h-period ahead forecast made at time $T+1$ gives

$$\boldsymbol{y}_{T+h}^{(T+1)} = \underbrace{\sum_{l=0}^{p-1} \Pi_l^{(\tau,h,T+1)} \boldsymbol{y}_{T-\tau-l}^{(T+1)}}_{\boldsymbol{dc1}_{T+h}^{(\tau,T+1,h)}} + \underbrace{\Pi_0^{(\tau,h,T+1)} \boldsymbol{c}}_{\boldsymbol{dc2}_{T+h}^{(\tau,T+1,h)}} + \\ + \underbrace{\sum_{l=-1}^{\tau-1} C_l^{(h,T+1)} B^{(T+1)} \boldsymbol{\epsilon}_{T-l}^{(T+1)}}_{\boldsymbol{sc1}_{T+h}^{(\tau,T+1,h)}} + \underbrace{\sum_{l=2}^{h} C_l^{(h,T+1)} B^{(T+1)} \boldsymbol{\epsilon}_{T+l}^{(T+1)}}_{\boldsymbol{sc2}_{T+h}^{(\tau,T+1,h)}}. \tag{18}$$

Note that both forecasts are written as a function of the data until $(T-\tau)$, rather than writing the forecast made at time $(T+1)$ as a function of data up to $T-\tau+1$. Note also that the structural shocks at time $T+1$ move from the future stochastic component $\boldsymbol{sc2}_{T+h}^{(\tau,T,h)}$ (i.e. $\boldsymbol{\epsilon}_{T+1}^{(T)}$) to the present stochastic component $\boldsymbol{sc1}_{T+h}^{(\tau,T+1,h)}$ (i.e. $\boldsymbol{\epsilon}_{T+1}$), since they were simulated for the forecast made at time T, but estimated for the forecast at time $T+1$.

With this setting, the forecast revisions for horizons $h = 2, .., H$ can be written as

$$\begin{aligned} \boldsymbol{y}_{T+h}^{(T+1)} - \boldsymbol{y}_{T+h}^{(T)} = \quad & \boldsymbol{dc1}_{T+h}^{(\tau,T+1,h)} - \boldsymbol{dc1}_{T+h}^{(\tau,T,h)} + \\ & + \boldsymbol{dc2}_{T+h}^{(\tau,T+1,h)} - \boldsymbol{dc2}_{T+h}^{(\tau,T,h)} + \\ & + \boldsymbol{sc1}_{T+h}^{(\tau,T+1,h)} - \boldsymbol{sc1}_{T+h}^{(\tau,T,h)} + \\ & + \boldsymbol{sc2}_{T+h}^{(\tau,T+1,h)} - \boldsymbol{sc2}_{T+h}^{(\tau,T,h)}. \end{aligned} \tag{19}$$

This equation illustrates to what extent the forecast revision is driven by an update in the estimate of the deterministic component (including the estimate of the unconditional mean of the model), a revision in the role attributed by the two forecasts to the shocks estimated between time $T-\tau+1$ and T, an effect associated with the shocks that hit at time $T+1$ relative to the value simulated in the forecast from time T, and a change in the role of future shocks over the remaining forecast horizon. All of these four components will reflect a potential change in the parameters or the structural shocks.[4]

Equation (19) shows the decomposition of the forecast revision. A similar decomposition holds with respect to the forecast error. Following equation (2), the data

[4]We acknowledge that the deterministic component could well capture revisions in the estimates of the structural shocks when τ is small.

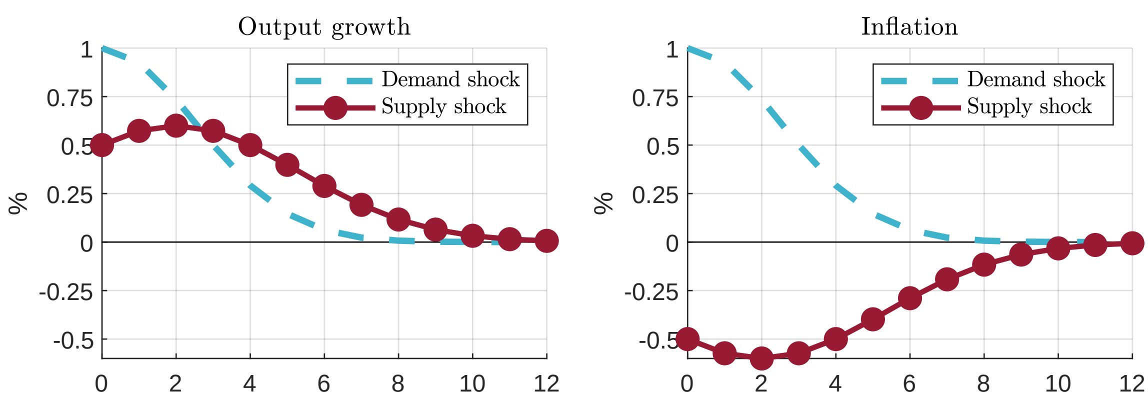
realization $\boldsymbol{y}_{T+1}$ can be decomposed into the deterministic component up to time $T-\tau$ and the role of the structural shocks from $T-\tau+1$ to $T+1$. Hence, similar to the forecast revisions, also forecast errors can be decomposed into the role attributed to the change in the deterministic component and the role of the subsequent structural shocks.

3 An illustration using simulated data on a bivariate model

We use a bivariate simulation to further illustrate the decomposition of forecast errors and revisions proposed in the paper.

We specify the data generating process as a bivariate SVAR model with a constant term and 12 lags. We use the model to generate data for the generic variables y_{1t} and y_{2t}, which are driven by shocks ϵ_{1t} and ϵ_{2t}. For simplicity, we will refer to the variables as output growth and inflation, and will view the structural shocks as pseudo demand and supply shocks. These interpretations are purely illustrative, as we work with simulated data.

Figure 1: Illustration: true impulse responses



To set the parameters of the model we follow the approach proposed by Canova et al. (2024). We first specify the true impulse responses of output growth and inflation to the demand and supply shocks. We then set the true parameters of the model equal to the SVAR parameters consistent with the true impulse responses. Figure 1 shows the true impulse responses of the model.[5] A positive one-standard-deviation demand

[5]We refer to Appendix A of the Online Appendix for the detailed discussion of the parametrization

shock (blue dashed lines) increases output growth and inflation on impact by 1%. The responses then slowly revert back to zero, reaching half of the impact effect three periods after the shock. By contrast, a positive one-standard-deviation supply shock (red dotted line) increases output growth on impact by 0.5% and decreases inflation by 0.5%. Contrary to demand shocks, supply shocks generate hump-shaped impulse responses that reach the peak effect 2 periods after the shock, at a value that is 20% higher than the impact effect. The SVAR parameters consistent with these impulse responses imply model stationarity. Last, we set the true constant terms of the model such that the model-implied unconditional mean for the pseudo output growth and inflation equals 1.5% and 2%, respectively.

We use the model to construct the following exercise. We simulate 200 periods of pseudo data, initializing the data at the unconditional mean of the model. To generate data, we randomly draw shocks from their distribution, except for the demand shock in the last five periods, which we set equal to one standard deviation. This generates a period of strong growth and elevated inflation, which serves as starting point of the exercise. Then, starting from period $T = 200$, we simulate an unconditional forecast until horizon $T+H = 220$, assuming zero future shocks. Last, we simulate data for period $T+1 = 201$ and then simulate a new unconditional forecast from the point of view of period $T+1$ over the forecast horizon $T+2 = 202$ to $T+H = 220$, still assuming zero future shocks. This framework implies forecast errors at time $T+1 = 201$, and a forecast revision from $T+2 = 202$ to $T+H = 220$. Last, we use the structural form of the model to help interpret the economic forces driving the forecast errors and revisions.

Following the discussion in the previous section, we set $\tau = 5$ in equations (17)-(18), and interpret the forecast errors and revisions as the sum of three components:

a) the difference in the role that the two forecasts associate with the deterministic component;

b) the difference in the role that the two forecasts associate with the shocks that hit the model-economy in the overlapping sample period between $T-\tau+1 = 196$ and $T = 200$;

c) the role that the forecast made at time $T+1 = 201$ associates with the shocks at time $T+1 = 201$, which the forecast at time T set to zero.

of the model.

3.1 Illustration in a simplified setting

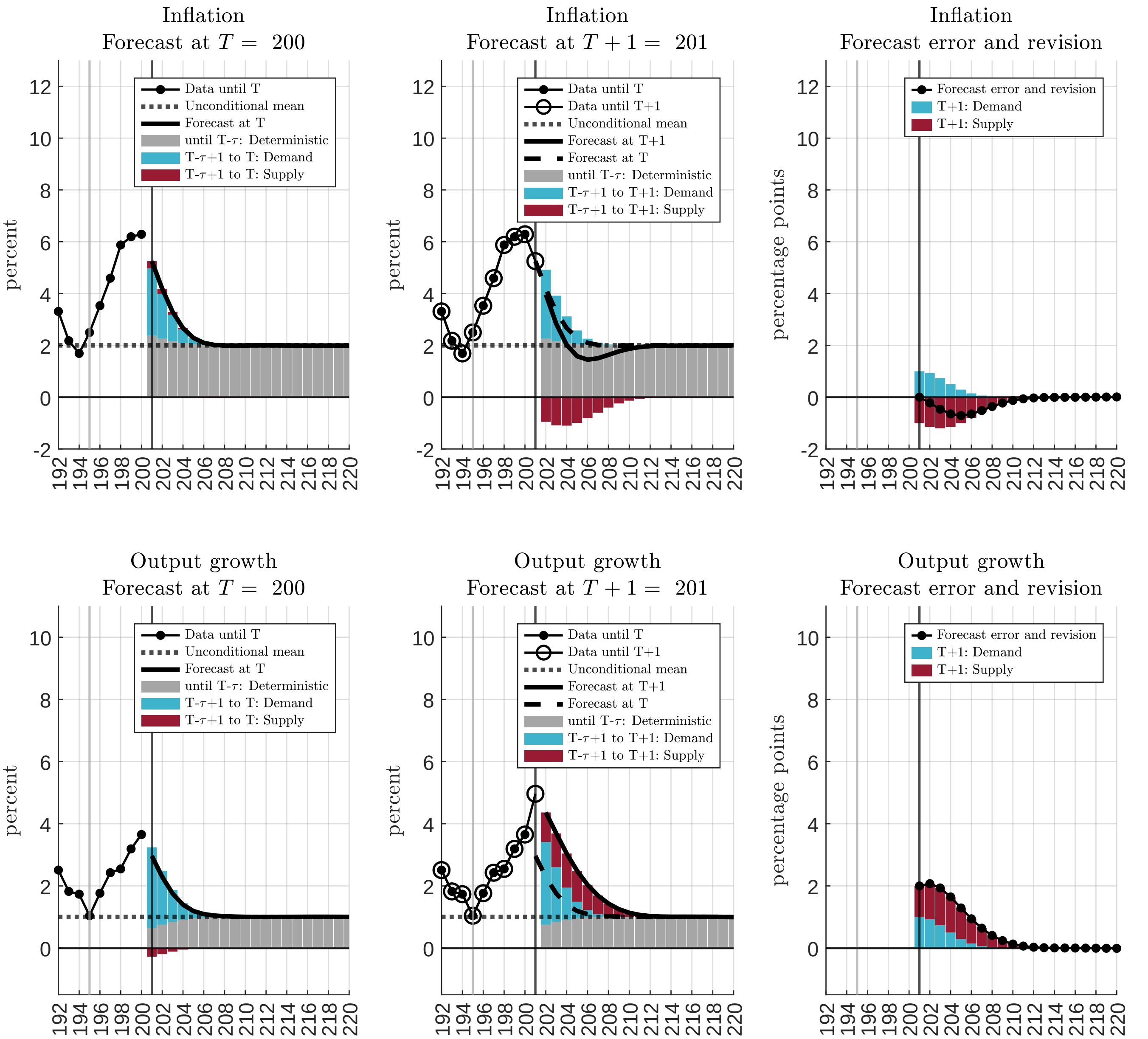
We begin from a case where we initially assume that no data revisions take place. This means that the data covering the period up to T is the same when forecasting at time T or at $T{+}1$. We also temporarily assume that the true parameters of the model are known when generating both forecasts, and that they do not change between the two periods. These are important assumptions because they imply that point a) and b) outlined above are null. We then generalize the simulation in the second part of this section. It is however important to stress that the analysis is conducted on the unconditional forecast in both the simplified and generalized settings. This implies that both forecasts are generated using the reduced form representation of the model, and we assume future zero shocks, which implies that the stochastic component $\boldsymbol{sc2}$ in equations (17)-(18) is zero.

Figure 2 reports the analysis for both inflation (top panel) and output growth (bottom panel). For both panels, the left plots show the data available until time T, the unconditional mean of the model, the forecast made at time T, and its decomposition into deterministic component and stochastic components for shocks between $T{-}\tau{+}1$ and T. The middle plots show the data available both until time T and $T{+}1$, the unconditional mean of the model, the forecast at time $T{-}1$, the new forecast made at time $T{+}1$, and the decomposition of the new forecast into deterministic component, and the stochastic component from shocks between $T{-}\tau{+}1$ and $T{+}1$. The right plots show the forecast error made at time $T{+}1$, the forecast revision until $T{+}H$, and the decomposition of the forecast error and revision. In all figures, the gray vertical line indicates $T{-}\tau$ while the vertical black line indicates $T{+}1$.

The left plots of Figure 2 show that at time T, the model predicts a slow decline of output growth and inflation towards the unconditional mean, with no undershooting relative to the long term. At time $T{+}1$ (middle plots) the new data turns out to be in line with the forecast for inflation, but 2% above the forecast for output growth. In addition, the new forecast outlines that inflation will temporarily *undershoot* the unconditional mean, and output growth will decline much less rapidly. The right-hand side plots display this graphically, showing no forecast error and a downward forecast revision for inflation, and a positive forecast error and positive forecast revision for output growth.

A purely reduced form approach to forecasting would provide very limited support to the interpretation of the forecast errors and forecast revisions. A researcher would not be able to go beyond stating that the new forecast at $T{+}1$ suggests an upward

Figure 2: Illustration with no data revision and true parameters



Note: The left-hand side panels show the forecast at time T (solid line), while the middle panels show the new forecast at time $T + 1$ (solid line) along with the one produced at time T (dashed line). Both forecasts are decomposed into the role of the different components up to time T and $T + 1$ respectively: demand shock (blue bars), supply shocks (red bars) and deterministic component (gray bars). The right-hand side panels plot the marginal difference between the forecasts, along with the contribution of each component.

revision for the forecast for output growth and a downward revision for the forecast for inflation, and that there was no forecast error for inflation and a positive forecast error for output growth. Any further narrative on the drivers of the positive forecast error for output growth and for the new undershooting of inflation would be speculative.

The methodology proposed in this paper offers a tool to derive a structural narrative of the forecast errors and forecast revisions. Let us now analyze the drivers of the two forecasts made at time T and $T+1$. The left-hand side panels of Figure 2 show that the forecast at time T is largely influenced by the multiple strong demand shocks that hit until period T, with their effects that propagate persistently through the system over the forecast horizon. As these shocks fade away, the forecast converges to the deterministic component of the model, which coincides with the unconditional mean in the long run. When we move forward by one period, the effects of the shocks up to T are still present. Yet, the forecast at time $T+1$ also reflects the realization of the new shocks that hit the economy at time $T+1$, which were assumed to be zero from the point of view of the forecast at time T.

In this illustration, the data realizations at $T+1$ were generated by simulating a positive one-standard-deviation demand shock and a positive (inflationary) two-standard-deviation supply shock. The joint effects of these shocks are noticeable in the middle panels. On output growth, both shocks are expansionary, and explain the strong forecast error between the two periods. Yet, for inflation, the fact that no forecast error is detected hides the fact that two opposite forces are playing out: an inflationary demand shock, and a deflationary supply shock. The undershooting of inflation predicted by the forecast made at time $T+1$ can now be rationalized as the effect of the supply shock: since supply shocks generate relatively weaker effects on impact but feature delayed effects via hump-shaped responses (see Figure 1), the large deflationary supply shock will indeed materialize in the medium term of the forecast, explaining the forecast revision and the undershooting of inflation.

The right-hand side plots of Figure 2 reveal that the forecast errors and forecast revisions are large due to the structural shocks that hit at time $T+1$. By contrast, no role is played by the revision in the deterministic component nor the role of the latest shocks before period $T+1$. This result is driven by the fact that no revisions apply to the data, and that the same (true) parameter values are used for both forecasts, hence the same estimates of the shocks. As a result, both forecasts predict the same role played by the constant term and by the data up to horizon $T-\tau$ (deterministic component). For the same reason, the two forecasts attribute the same role to the shocks between time $T-\tau+1$ and T. The only factor that differs is the treatment of period $T+1$. The forecasts made at time T assumes zero shocks at $T+1$, which hence play no role over the forecast. The forecast made at time $T+1$ estimates the shocks at time $T+1$ from the data, hence these shocks will be a driving forces of the variables over the forecast horizon.[6]

3.2 Illustration in a generalized setting

We conclude the illustration by bringing into the discussion the more realistic scenario in which the parameters are estimated, and the data can be subject to revisions from one forecast to the other. This has important consequences, as now the effects of the shocks up to time T can differ between the forecast at time T and the one at time $T+1$, with the marginal effects that can influence the new forecast path. This happens because different data over the past may lead the model to a different interpretation of the underlying shocks, and therefore could affect parameter estimates. Moreover, the latter can also change in the absence of data revisions, and solely with the new data realization at time $T+1$.[7]

We start from the original data at time T and add noise, modeled as the realization of independent Normal random variables with standard deviation set equal to 0.05. We then start from the original data until $T+1$ and subject it to noise, drawn in the same way as for time T. We use both datasets to estimate the reduced form parameters using Ordinary Least Squares. Last, we estimate the structural impact effect of the shocks by applying to the estimated Cholesky decomposition of the reduced form variances the true orthogonal matrix associated with the data generating process. Forecasts and decompositions are then generated using the parameter estimates and the data subject to noise.[8]

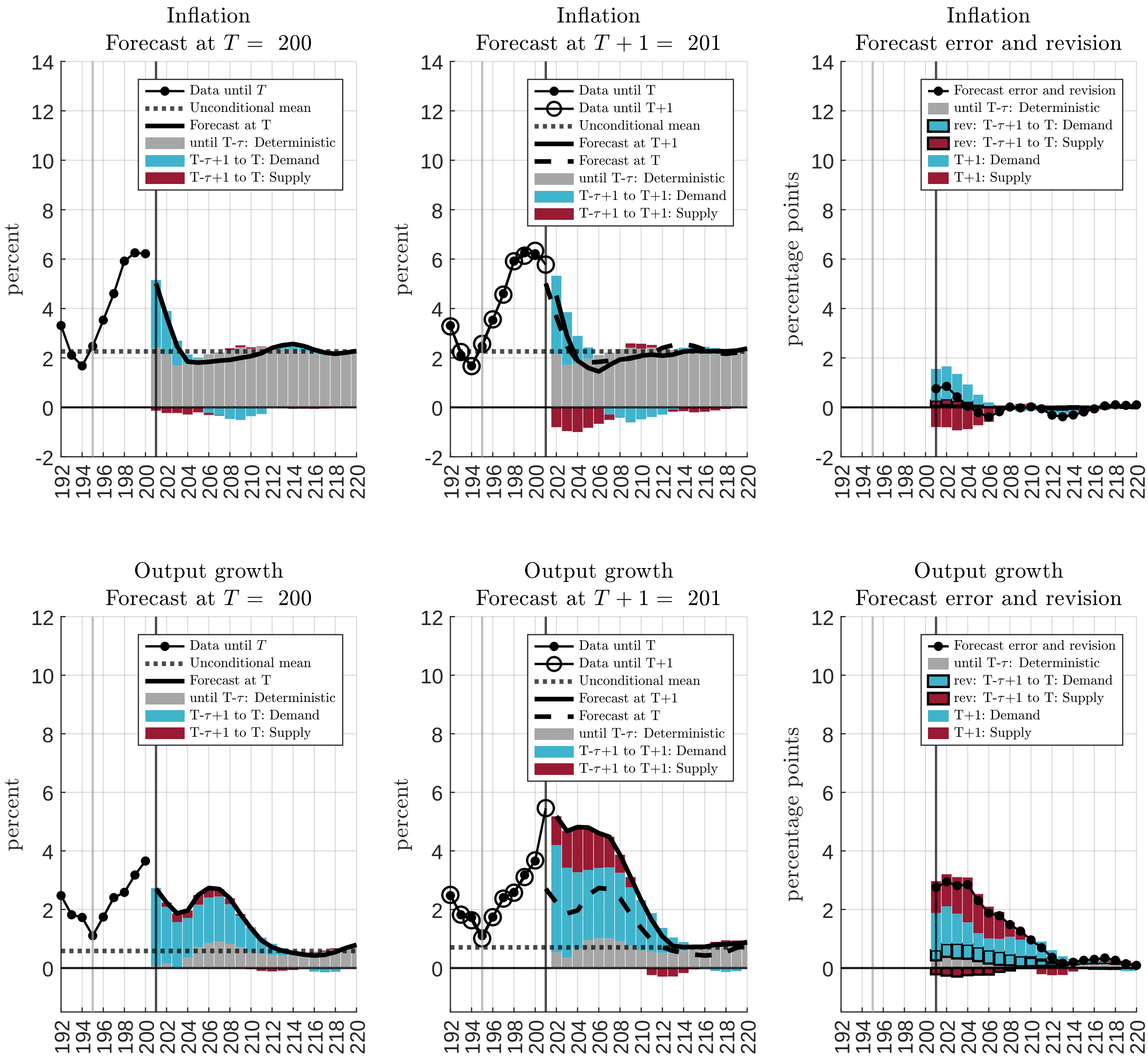
The analysis with data revision is reported in Figure 3. While the narrative of the forecasts and the decompositions is similar to Figure 2, a few differences are worth highlighting. First, the forecasts made at time T and at $T+1$ could indeed reflect

[6] Figure A-1 in the Online Appendix breaks down the stochastic components of each forecast into the contribution from the shocks of each period (composite impulse responses).

[7] In principle, one could disentangle the role of data revisions and the new parameter estimates by not re-estimating the model at time $T+1$, thus using a model with the parameters estimated at time T but on new data realization.

[8] We follow this approach in order to avoid entering issues related to the identification of the shocks, which we view as relevant only in applications and not strictly important for the simulation exercise from this section.

Figure 3: Illustration with data revision and estimated parameters



Note: The left-hand side panels show the forecast at time T (solid line), while the middle panels show the new forecast at time $T + 1$ (solid line) along with the one produced at time T (dashed line). Both forecasts are decomposed into the role of the different components up to time T and $T + 1$ respectively: demand shock (blue bars), supply shocks (red bars) and deterministic component (gray bars). The right-hand side panels plot the marginal difference between the forecasts, along with the contribution of each component.

differences in the estimated unconditional mean of the model, although the effect is only limited in the proposed illustration, as seen from the very small gray area on the right plots. Second, the fact that the two forecasts use different parameter estimates and hence different estimates of the shocks imply that the forecasts can associate different roles to the shocks in the period between $T-\tau+1$ and T. For instance, the forecast made at time $T+1$ interprets the demand shocks until T as being more expansionary and the supply shocks as being more contractionary compared to the forecast at time T. The effects of the revised shocks on inflation and output growth are reported in the right-hand side panels by the highlighted bars.[9]

4 An application to the surge of inflation in 2022

In this section, we use our methodology in a SVAR model for the UK economy. More precisely, we apply the framework explained in section 2 to analyze the period of high inflation that followed the Covid-19 pandemic. We first describe the specification of the model, including the data, identification of the shocks, and estimation procedure. We then outline the real-time forecast exercise, and discuss the results on forecast errors and revisions. We keep the model parsimonious and tractable, as the main intent of this section is to showcase to the reader the possible use and benefits of the methodology.

4.1 Model specification, identification, and estimation

We estimate an SVAR model of the form described in equation (1). The model includes four variables: (i) the UK policy rate captured by the Bank rate; (ii) Real GDP; (iii) the consumer price index; (iv) Real oil prices. Except for the Bank rate, all variables enter the model in log difference, in order to ensure stationarity. The frequency of the data is quarterly, and the full sample covers the period from 1992Q1 to 2025Q3. The data are downloaded from the Office of National Statistics.

We identify four structural shocks, so that data volatility is fully explained by the identified shocks in our system. We use sign restrictions (Uhlig, 2005; Baumeister and Hamilton, 2015; Arias et al., 2018) to identify generic demand and supply shocks, along with a monetary policy shock and an energy shock. Restrictions, reported in Table B-1

[9] Figure A-2 in the Online Appendix breaks down the stochastic components of each forecast into the contribution from the shocks of each period (composite impulse responses).

of the Online Appendix, are rather standard in the literature, and are introduced only on the impact effect of the shocks, with no restrictions on the future horizons nor on the contemporaneous relationship among variables.

We follow Giannone et al. (2015) and we use Bayesian shrinkage to estimate the model. We set $p = 4$ as the number of lags in the model and use a Minnesota prior on the autoregressive parameters. We also add a single-unit-root prior to discipline the deterministic component, as in Bergholt et al. (2024). To deal with the volatility over the Covid period, we add Covid-19 dummies for the quarters from 2020Q1 to 2021Q2 following the pandemic prior approach proposed by Cascaldi-Garcia (2022). Finally, we apply the methodology proposed by Arias et al. (2018) to identify the four structural shocks.

4.2 Results from forecast analysis

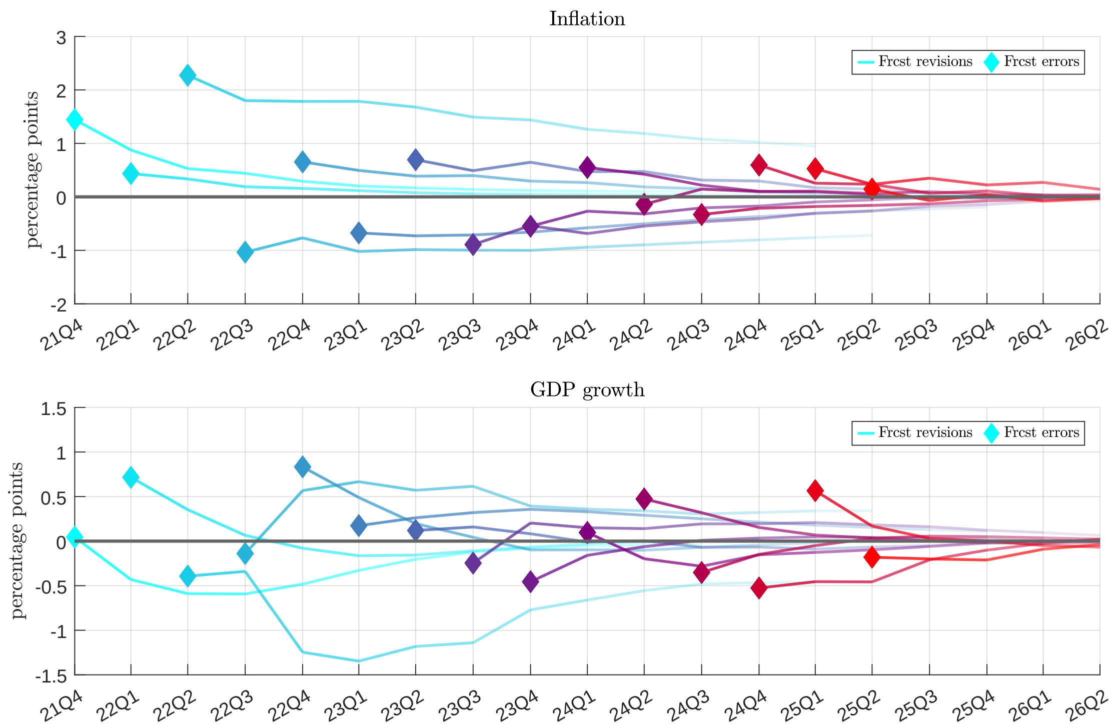
Our goal is to replicate on real data what described in section 3 on simulated data. To do this, we conduct a real-time forecast exercise and iterate over each of the sixteen quarters that go from 2021Q4 to 2025Q3. For each quarter, which we label as time T, we estimate the model with data from 1992Q2 up to time T and we produce an unconditional forecast with zero future shocks (equation 16), simulating the forecast H-steps ahead with $H = 12$.[10] We then move to the next quarter, we re-estimate the model using data from 1992Q2 to $T{+}1$, simulate the new forecast made at $T{+}1$, and compute the forecast errors and forecast revisions relative to the previous quarter. Importantly, we conduct the exercise using the vintages of the data available at each new quarter, so that we can also account for the possible role of data revisions over time. Last, for each quarter we apply the forecast decomposition outlined in section 2, setting $\tau = 12$. This implies that we decompose each forecast into the role of the data up to three years before the forecast and into the subsequent structural shocks.[11]

Before discussing our structural forecast decompositions, we find it helpful to document the *reduced form* results of this exercise. Figure 4 reports the sequence of forecast errors and forecast revisions implied by the model for the QoQ inflation (top

[10]We choose $H = 12$ as it coincides with the 3-years ahead forecast horizon usually analyzed by central banks.

[11]More precisely, we simulate the forecasts using data until 2021Q4 and then until 2022Q1, and decompose both as a function of the data until 2018Q4 and the subsequent shocks. Then, we simulate the forecast at 2022Q2, and decompose the forecasts at 2022Q1 and 2022Q2 as a function of the data up to 2019Q1 and the subsequent shocks. We continue until the forecasts for 2025Q2 and 2025Q3, which are decomposed as a function of the data up to 2022Q2 and the subsequent shocks.

Figure 4: Sequence of estimated mean forecast errors and forecast revisions

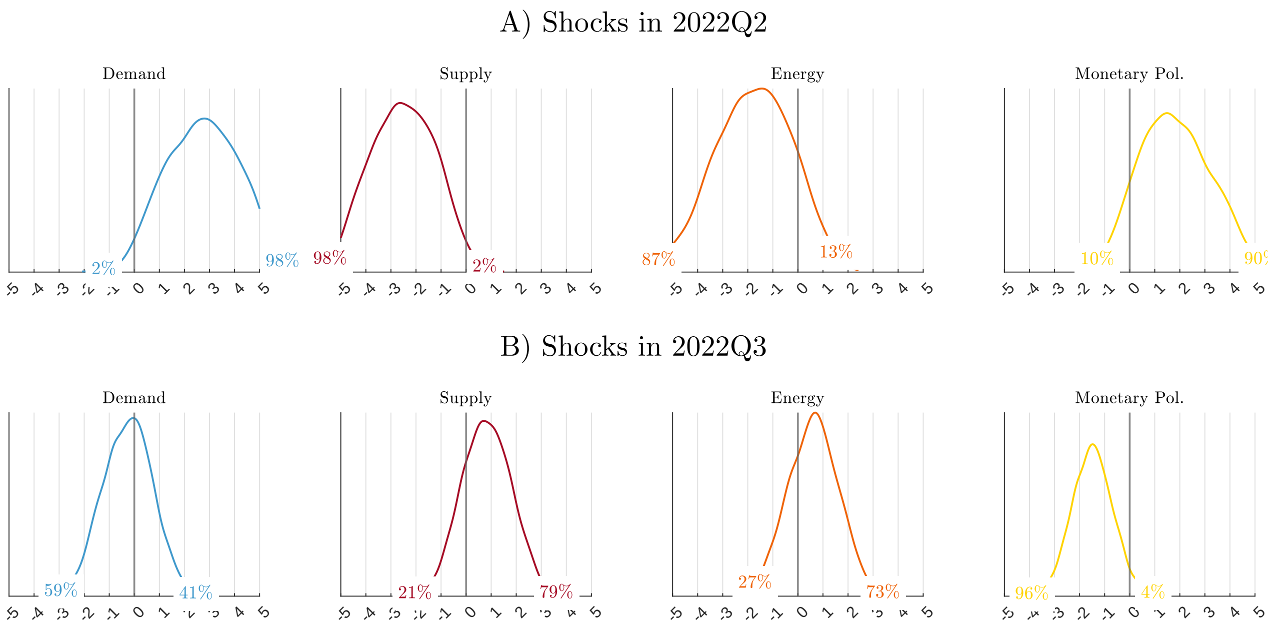


Note: Diamonds indicate forecast errors for each forecast produced in a specific quarter, while lines the related forecast revisions.

panel) and QoQ GDP growth (lower panel). The diamond in each period t reports the forecast error for that period, while the line that starts from each diamond reports the subsequent forecast revision.[12] A few findings are immediately visible from the figure. First, forecast errors and revisions over 2022 and 2023 are sizable for both GDP and inflation, and bigger compared to subsequent periods. This is not surprising, and many in the literature have documented sizable forecast errors in this period (see, for instance, Giannone and Primiceri, 2024). Second, the comovement in the forecast errors and forecast revisions for inflation and GDP growth can vary a lot over the periods. For example, in 2022Q2 – the quarter associated with the Russian invasion of Ukraine – forecast errors comoved strongly and negatively, with a subsequent marked forecast revision. By contrast, in 2024Q4 the forecast errors comoved positively, with a much more moderate revision in the subsequent forecast. Third, in some periods,

[12] The forecast error at time t is reported as the difference between the first data realization that became available at time t for time t data and the forecast made for that period at $t-1$. The forecast revision is reported as the difference between the newly formed forecast made at time t and the previous forecast made at $t-1$.

Figure 5: Estimated structural shocks



Note: The top panel shows the marginal posterior distribution of shocks for 2022Q2 as estimated when data up to 2022Q2 became available. The bottom panel shows the same for 2022Q3. Each plot reports the Positive values mean that the shocks are expansionary. This means that GDP increases for all the shocks analyzed, while inflation increases with demand and monetary policy shocks, and decreases with supply and energy shocks, see Figure B-4.

small forecast errors can be associated with large forecast revisions, as, for instance, for GDP growth in 2022Q1 and 2022Q3.

It would be tempting to conclude from Figure 4 in favour of a demand-side narrative for periods when the forecast errors of inflation and GDP comoved positively, and of a supply-side narrative for periods when the forecast errors comoved negatively. However, this approach would reveal only a partial story. First, it does not explore the fact that an overall positive comovement in forecast errors might still hide multiple shocks exerting very different effects on the variables. Second, it would be inconclusive for periods in which the forecast error is close to zero for only one of the variables, for instance for 2021Q4 or 2022Q3. Third, as explained in section 2, forecast errors and forecast revisions are affected not only by the contemporaneous shocks, but also by possible revisions of the shocks taking place between forecasts.

To better highlight our first point, let us focus on 2022Q2 and 2022Q3. Figure 5 shows the posterior marginal distribution of the estimated structural shocks in periods 2022Q2 (top panel) and 2022Q3 (bottom panel). The distributions refer to shocks estimated using the earliest vintage of the data, hence data up to 2022Q2 (top panel) or up to 2022Q3 (bottom panel).[13] As suggested by Figure 4, the negative comovement in the forecast errors for inflation and GDP growth in 2022Q2 suggests the presence of supply-side shocks. While both the supply shock and the energy shock from the model were estimated to be large and deflationary, the model also estimates a (more moderate) expansionary demand shock and monetary policy shock. As for 2022Q3, the negligible forecast error documented in Figure 4 for GDP growth is masking some underlying shocks that hit the economy, such as a positive supply shock and a contractionary monetary policy shock.

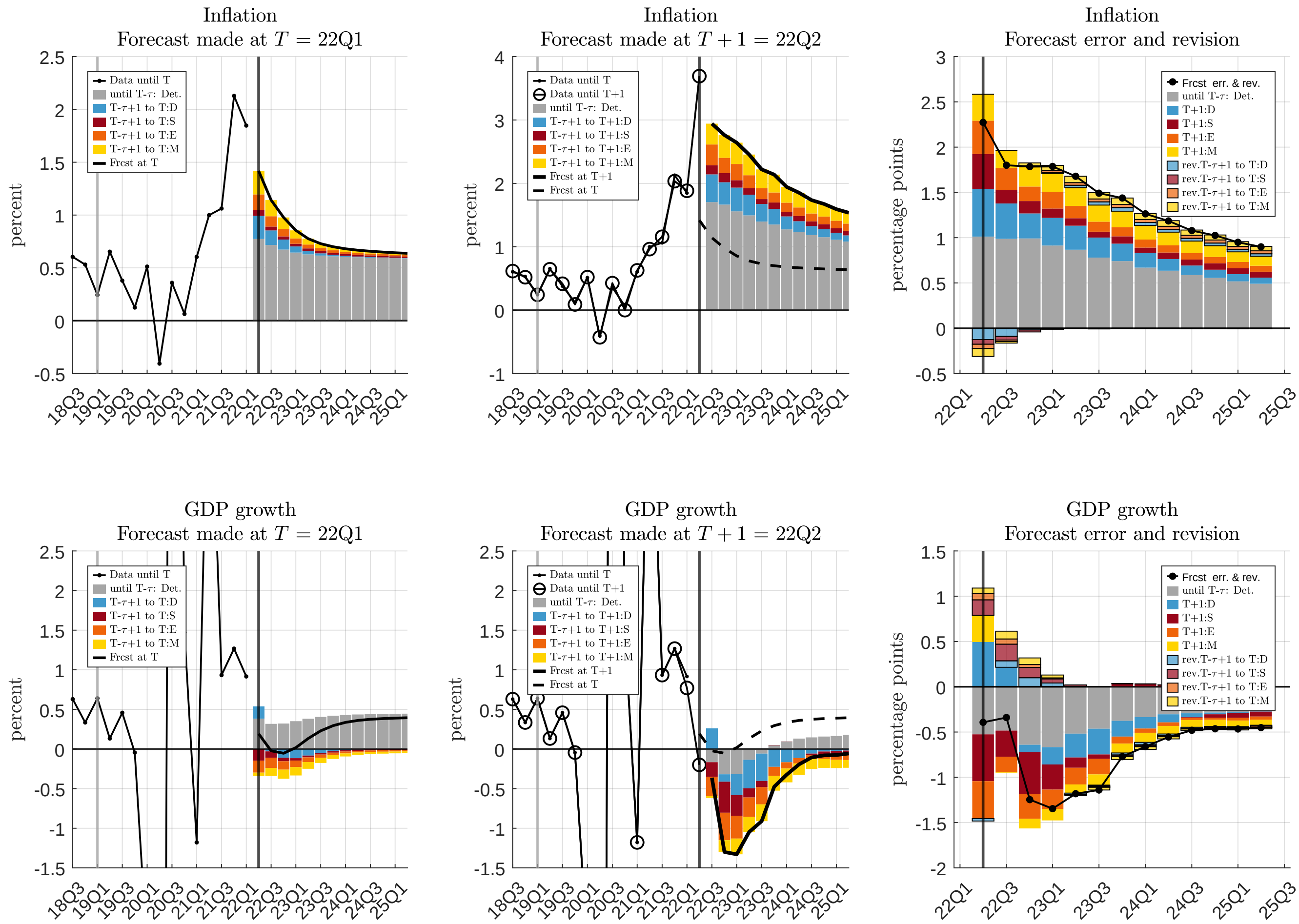
As outlined in section 2, latest shocks is only part of the story when it comes to explain forecast errors and revisions, as the forecast can be viewed as the sum of three components: (1) the role of changes in the unconditional mean, or, more generally, the deterministic component; (2) the role associated with the revision of the shocks estimated in the last three years before period T; and (3) the role associated with the latest shocks that were estimated for $T{+}1$, which were assumed to equal zero from the point of view of time T. To better show this, we now show the full decomposition. Figure 6 reports the results for 2022Q2. From Figure 4 we already know that this quarter is associated with the largest forecast error for inflation, which came in 2.3 percentage points higher with respect to the forecast produced in 2022Q1, and with a negative forecast error for GDP equal to -0.4.

We begin by describing the *absolute* forecasts shown in the left and middle plots of Figure 6. The figure shows the pointwise mean forecasts and pointwise mean of each of its components. In 2022Q1 (left-hand side panels) the model is predicting GDP growth to reach 0% around 2022Q4, and inflation to reach 1.5%, before moving gradually to 0.5% towards the end of the horizon. Our decomposition suggests that the inflation forecast is elevated due to a mix of negative supply (in red) and energy (in orange) shocks, but also expansionary demand (in blue) and monetary policy (in yellow) shocks. The *absolute* narrative is similar for GDP growth, with negative supply and energy shocks causing GDP growth to be weak. At the same time, previously positive demand and monetary shocks turn negative on growth, contributing to the weak forecast.[14] As we reach $T{+}1 = 2022Q2$, the data is found to have been revised, as can be seen by the difference between the dotted and the circled lines in the mid-

[13]See Figure B-5 in the Online Appendix for the distributions of the shocks in 2022Q2 and 2022Q3 estimated using subsequent three quarters of data vintages.

[14]The apparent puzzle of the role of monetary and demand shocks between Inflation and GDP growth depends on the persistency of the shocks, which is linked to the IRFs reported in Figure B-4.

Figure 6: Forecast analysis for 2022Q2

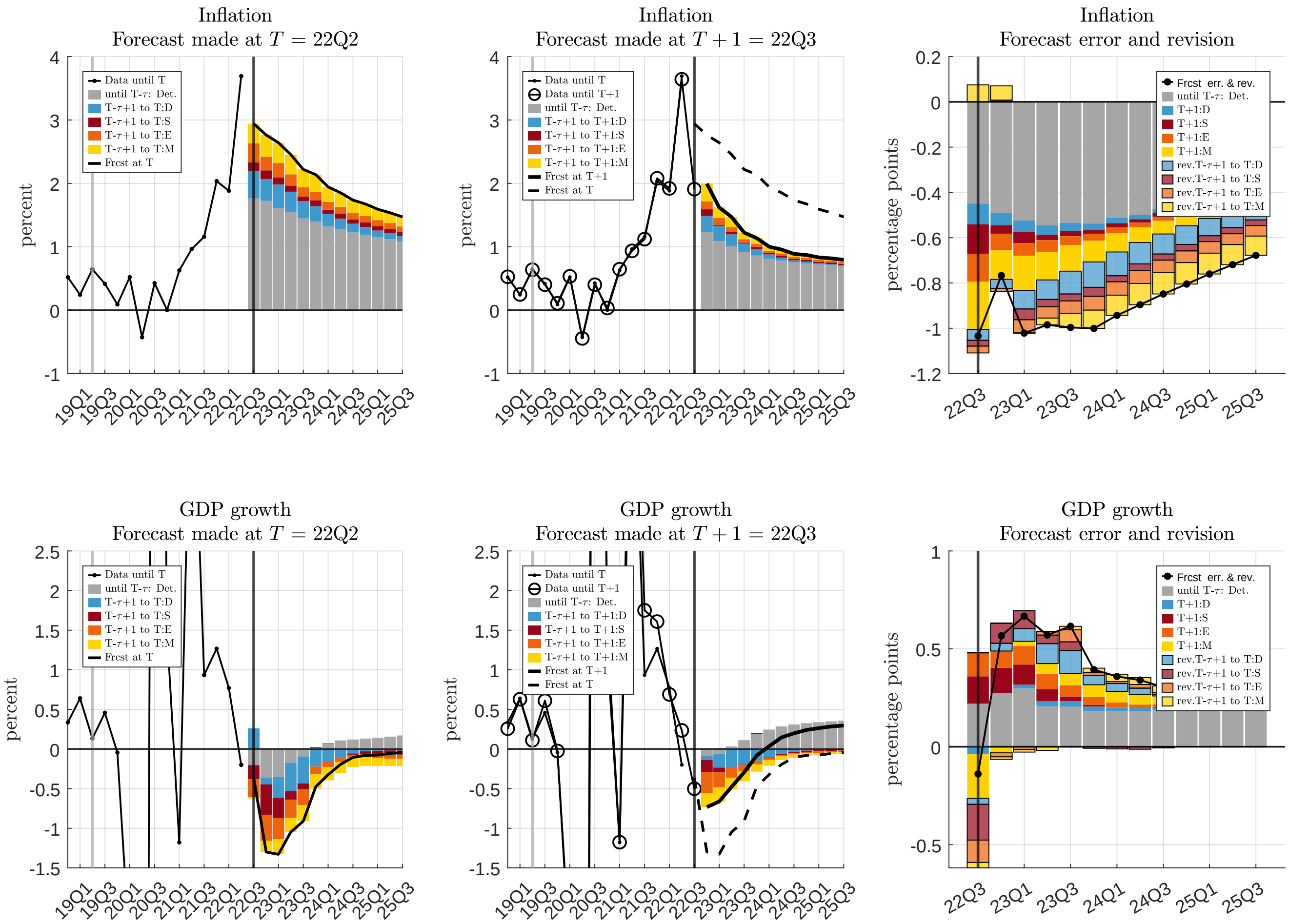


Note: The left-hand side panels show the forecast at time T (solid line), while the middle panels show the new forecast at time $T+1$ (solid line) along with the one produced at time T (dashed line). Both forecasts are decomposed into the role of the different components up to time T and $T+1$ respectively: demand (blue bars), supply (red bars), energy (orange bars), monetary policy shocks (yellow bars) and deterministic component (grey bars). The right-hand side panels plot the marginal difference between the forecasts, along with the contribution of each component.

dle panels. Data in 2022Q2 came in almost -0.5 percentage points lower than what predicted in 2022Q1 for GDP, and almost 2.3 percentage points higher for inflation. On the latter, the new forecast produced at time $T+1$ is overall higher for inflation, but lower for GDP. In *absolute* space, the model interprets the elevated path for the inflation forecast similarly to time T, with a mix of negative supply and energy shocks, and positive demand and monetary policy shocks all pushing up on inflation.

The left-hand side panels complement these results by plotting the decomposition

Figure 7: Forecast analysis for 2022Q3



Note: The left-hand side panels show the forecast at time T (solid line), while the middle panels show the new forecast at time $T+1$ (solid line) along with the one produced at time T (dashed line). Both forecasts are decomposed into the role of the different components up to time T and $T+1$ respectively: demand (blue bars), supply (red bars), energy (orange bars), monetary policy shocks (yellow bars) and deterministic component (gray bars). The right-hand side panels plot the marginal difference between the forecasts, along with the contribution of each component.

of the marginal change between the two forecasts, and thus help us interpret what drives the forecast errors and revisions between quarters. As shown by the right-hand side panels in Figure 6, only around 1 percentage points of the 2.6 forecast error for inflation was driven by the supply-side (supply and energy) shocks estimated for 2022Q2. The rest of the forecast error is interpreted by the model as a combination of demand-side (demand and monetary policy) shocks and an upward revision in the role of the deterministic component of the model. The revision of the role of previous shocks over the last three years before the forecast, instead, is found to play a more marginal role in driving the forecast errors and revisions, except for a limited downward pressure in the very short run. The interpretation for GDP growth is similar, with the negative forecast error for GDP growth and subsequent downward forecast revision being driven by a mix of contractionary supply-side shocks and expansionary demand-side shocks, as well as revisions in the deterministic component of the model.

This analysis can be conducted for every different round, and provides a consistent way to study the drivers of the forecast and to form a narrative on the reasons behind its change in between periods. Figure 7 reports the analysis for 2022Q3. The left-hand side panels show the forecast associated with 2022Q2 and its decomposition, while the middle panels report the forecast computed in 2022Q3. Relative to 2022Q2, the forecast errors for inflation is negative and the revision for inflation is downwards. As shown in the right-hand side panels, one of the factors contributing to this finding is a mix of mildly deflationary energy and supply shocks, along with a minor restrictive monetary policy shock. However, contrary to Figure 6, this time a big role is also played by the revised shocks, as now the model reviews the previous shocks to be less inflationary.[15] Overall, it is interesting to notice that the model suggests an important role of demand and monetary policy shocks to explain forecast errors and revisions, a result in line with what found for the US and the Euro Area by Giannone and Primiceri (2024). The increasing role played by demand-type shocks are also confirmed by the next quarter, where the inflation forecast is revised up mainly due to demand shocks, see Figure B-6 in the Online Appendix.

We conclude the analysis with a word of caution, remarking that careful consideration is required in the model specification to produce accurate forecasts and credibly identified shocks. We stress that the main purpose of this application is not to validate the small-scale model described in this section, but to show the relation between forecast errors, forecast revisions, and structural shocks, describing how the framework from this paper can be applied quarter-by-quarter.

5 Conclusions

This paper shows that the structural representation of a VAR model can offer a way to derive a narrative for forecast errors and forecast evaluations in terms of structural shocks even when the forecasts of interest are unconditional reduced form forecasts.

[15] See also Figure B-8 in the Online Appendix, which report the revisions of the shocks.

Since being able to explain the forecasts plays a key role in forecasting – especially in policy institutions – we view the method proposed in the paper as a useful new entry to the toolkit of time series methods for macroeconomics.

The methodology proposed in the paper decomposes forecast errors and forecast revisions as a function of three components: a) changes in what the paper refers to as the estimated deterministic component of the model, which typically captures the initial conditions and the unconditional mean of the model; b) the role associated with the estimated shocks in the periods leading up to the forecasts; and c) the role of the latest shocks that are estimated after the realization of the new data.

We first show our methodology by using simulated data in a bivariate VAR model. We then apply our method to the UK economy and study forecast errors and revisions in the aftermath of the Russian invasion of Ukraine. We show that the strong upward revision in the inflation forecast in 2022Q2 was driven not only by contrationary supply and energy shocks that hit in 2022Q2, but also by expansionary demand and monetary policy shocks, along with an overall upward revision of the estimated unconditional mean pinning down the long run trend of inflation estimated by the model.

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Online Appendix for "Structural forecast analysis"

Davide Brignone and Michele Piffer

A Additional material for the simulation exercise in section 3

The parameter values of the data generating process are set by first specifying the true impulse responses over 12 horizons. Following Canova et al. (2024), we use the following formulation of the Gaussian basis function for each shock j and variable i:

$$\bar{\psi}_{ij,h} = a_{ij} \cdot e^{-\left(\frac{(h-b_{ij})^2}{c_{ij}^2}\right)+\frac{b_{ij}^2}{c_{ij}^2}}. \tag{A-1}$$

The function allows us to span $H+1$ dynamic responses with only a handful of parameters: a_{ij} captures the impact effect of shock j on variable i, b_{ij} corresponds to the horizon at which the peak effect is reached, and equals 0 if no hump-shaped response is desired, c_{ij} controls for the persistence of the response. Equation (A-1) extends the specification by Barnichon and Matthes (2018).

We specify $a_{11} = a_{21} = 1$, $a_{12} = 0.5$ and $a_{22} = 0.5$. Hence, a one standard deviation positive shock to the first shock increases both variables by 1, while a one standard deviation positive shock to the second shock increases the first variable by 0.5 and decreases the second variable by 0.5. We then set $b_{11} = b_{21} = 0$ and $a_{12} = a_{22} = 0$, so that the responses to the first shock generate no hump-shaped patterns, while the second shock generates peak effects two periods after the shock. Last, we set c_{ij} so that the response to the first shock reaches 0.5 three periods after the shock, while for the second shock it leads to a peak effect that sits 20% above the impact effect, in absolute value.

The implied impulse responses are shown in Figure 1 of the paper. We then use the method by Canova et al. (2024) to compute the following parameters of a SVAR model with 12 lags:

$$B = \begin{pmatrix} 1 & 0.1 \\ 1 & -0.5 \end{pmatrix}, \tag{A-2}$$

$$\Sigma = \begin{pmatrix} 1.25 & 0.75 \\ 0.75 & 1.25 \end{pmatrix}, \tag{A-3}$$

$$Q = \begin{pmatrix} 0.8944 & 0.4472 \\ 0.4472 & -0.8944 \end{pmatrix}, \tag{A-4}$$

$$\Pi_1 = \begin{pmatrix} 1.0362 & -0.1103 \\ -0.1103 & 1.0362 \end{pmatrix}, \tag{A-5}$$

$$\Pi_2 = \begin{pmatrix} -0.1185 & -0.0039 \\ -0.0039 & -0.1185 \end{pmatrix}, \tag{A-6}$$

$$\Pi_3 = \begin{pmatrix} -0.0825 & 0.0154 \\ 0.0154 & -0.0825 \end{pmatrix}, \tag{A-7}$$

$$\Pi_4 = \begin{pmatrix} -0.0420 & 0.0227 \\ 0.0227 & -0.0420 \end{pmatrix}, \tag{A-8}$$

$$\Pi_5 = \begin{pmatrix} -0.0115 & 0.0157 \\ 0.0157 & -0.0115 \end{pmatrix}, \tag{A-9}$$

$$\Pi_6 = \begin{pmatrix} 0.0043 & 0.0027 \\ 0.0027 & 0.0043 \end{pmatrix}, \tag{A-10}$$

$$\Pi_7 = \begin{pmatrix} 0.0086 & -0.0061 \\ -0.0061 & 0.0086 \end{pmatrix}, \tag{A-11}$$

$$\Pi_8 = \begin{pmatrix} 0.0066 & -0.0072 \\ -0.0072 & 0.0066 \end{pmatrix}, \tag{A-12}$$

$$\Pi_9 = \begin{pmatrix} 0.0026 & -0.0035 \\ -0.0035 & 0.0026 \end{pmatrix}, \tag{A-13}$$

$$\Pi_{10} = \begin{pmatrix} -0.0007 & 0.0004 \\ 0.0004 & -0.0007 \end{pmatrix}, \tag{A-14}$$

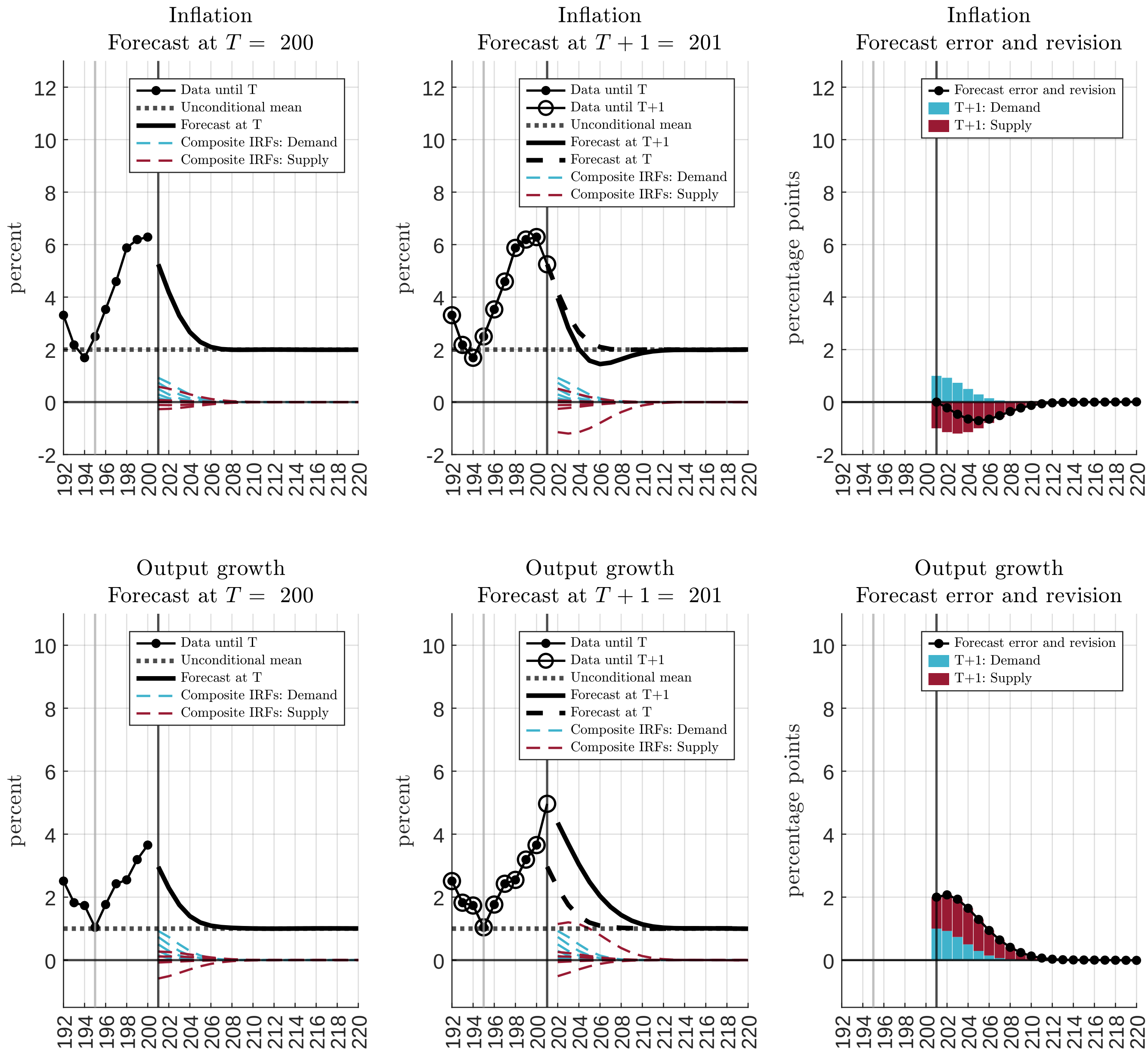
$$\Pi_{11} = \begin{pmatrix} -0.0020 & 0.0022 \\ 0.0022 & -0.0020 \end{pmatrix}, \tag{A-15}$$

$$\Pi_{12} = \begin{pmatrix} -0.0016 & 0.0018 \\ 0.0018 & -0.0016 \end{pmatrix}. \tag{A-16}$$

Last, the constant term were computed as

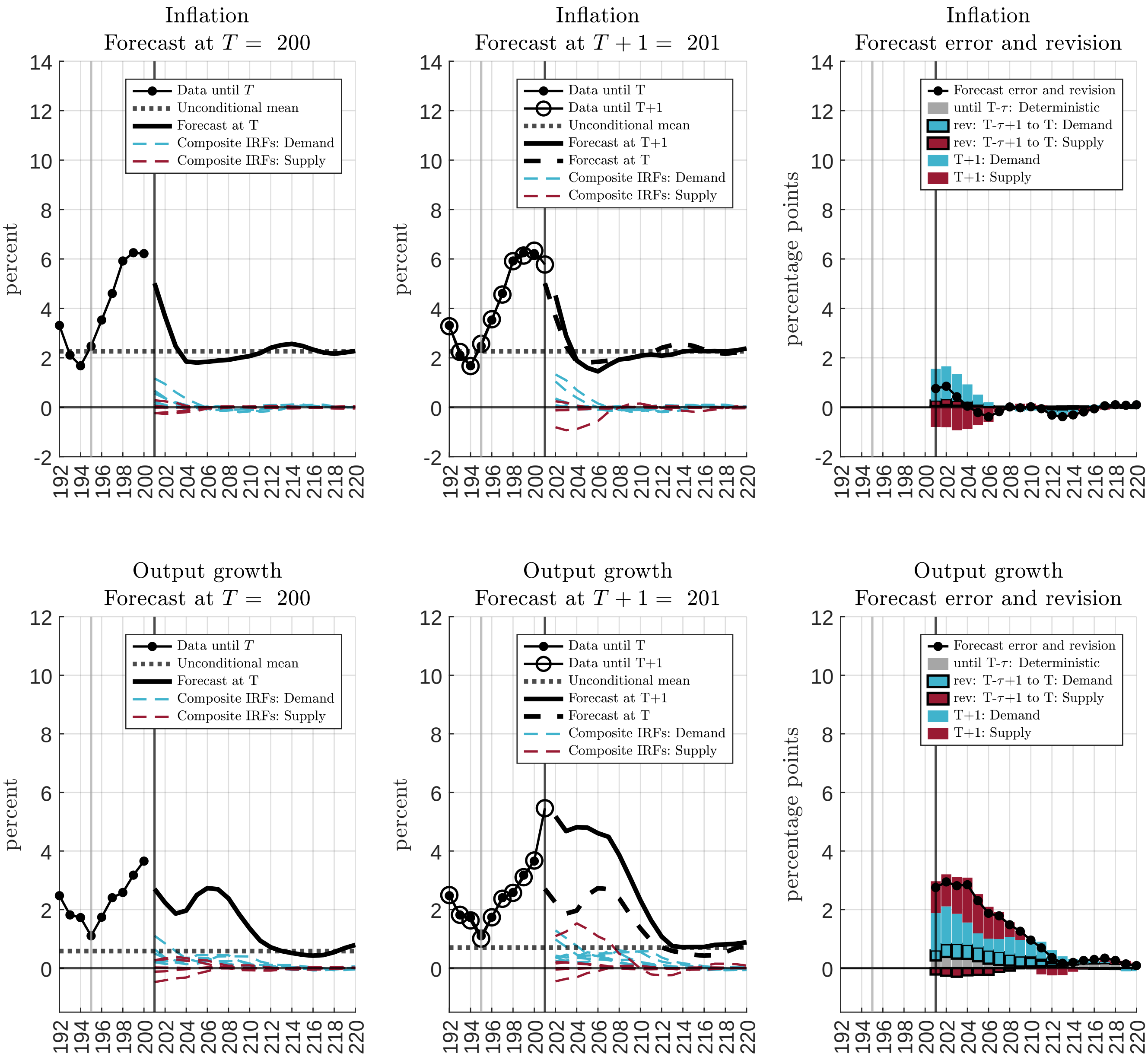
$$\boldsymbol{c} = \begin{pmatrix} 0.3409 \\ 0.4714 \end{pmatrix}. \tag{A-17}$$

Figure A-1: Illustration with no data revision and true parameters: composite impulse responses



Note: The dashed blue and red lines in the left and middle plots show the individual composite impulse responses for the shocks in each period from $T-\tau+1$ to either T (left plots) or $T+1$ (middle plots). By contrast, Figure 2 in the paper shows the pointwise sum across composite impulse responses.

Figure A-2: Illustration with data revision and estimated parameters: composite impulse responses



Note: The dashed blue and red lines in the left and middle plots show the individual composite impulse responses for the shocks in each period from $T-\tau+1$ to either T (left plots) or $T+1$ (middle plots). By contrast, Figure 2 in the paper shows the pointwise sum across composite impulse responses.

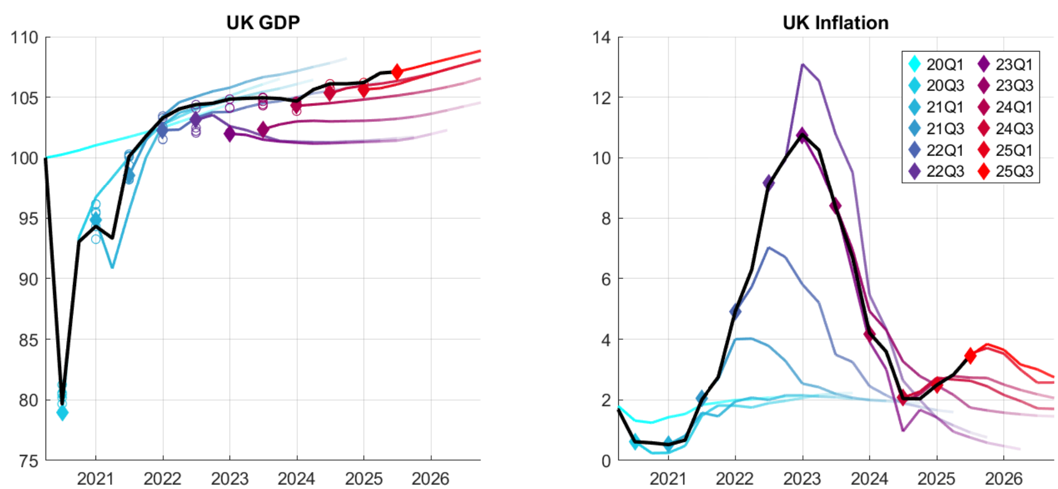
B Additional material for the application in section 4

Table B-1: Identifying restrictions

	Demand	Supply	Energy	Monetary
Bank rate	+			−
Real GDP growth	+	+	+	+
Inflation	+	−	−	+
Real oil prices	+	+	−	+

Note: The rows report model variables, while the columns document the identified shocks. Sign restrictions are introduced only on the impact effect of the shocks.

Figure B-3: Forecast revisions for UK GDP and UK Inflation



Note: Solid lines depict latest data for both UK GDP and UK inflation. The diamonds show the first nowcast for a specific quarters, while the colored shaded lines the related forecast. Finally, the circles exhibit the series of data revisions for a specific quarter.

Figure B-4: IRFs estimated for 2022Q2

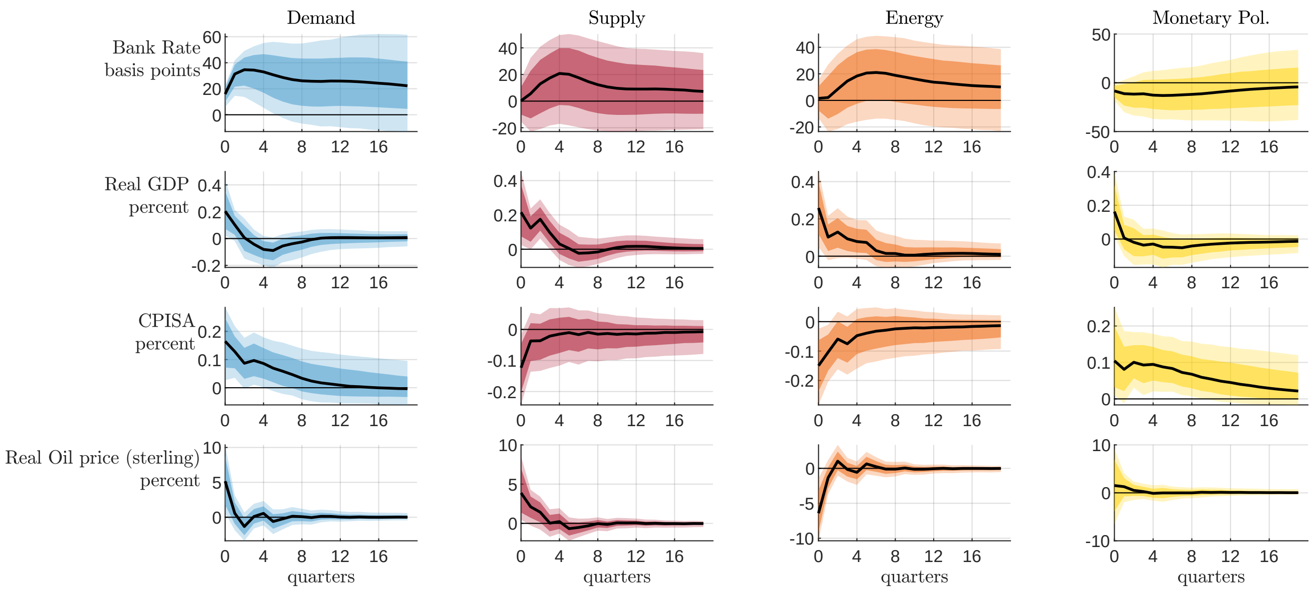
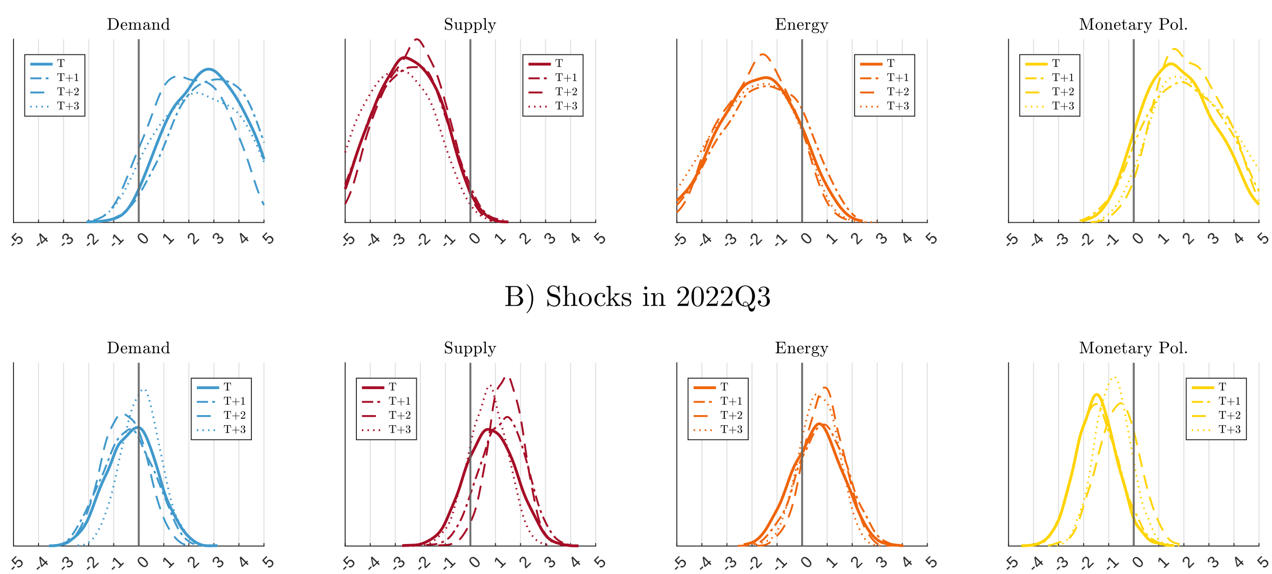


Figure B-5: Estimated structural shocks

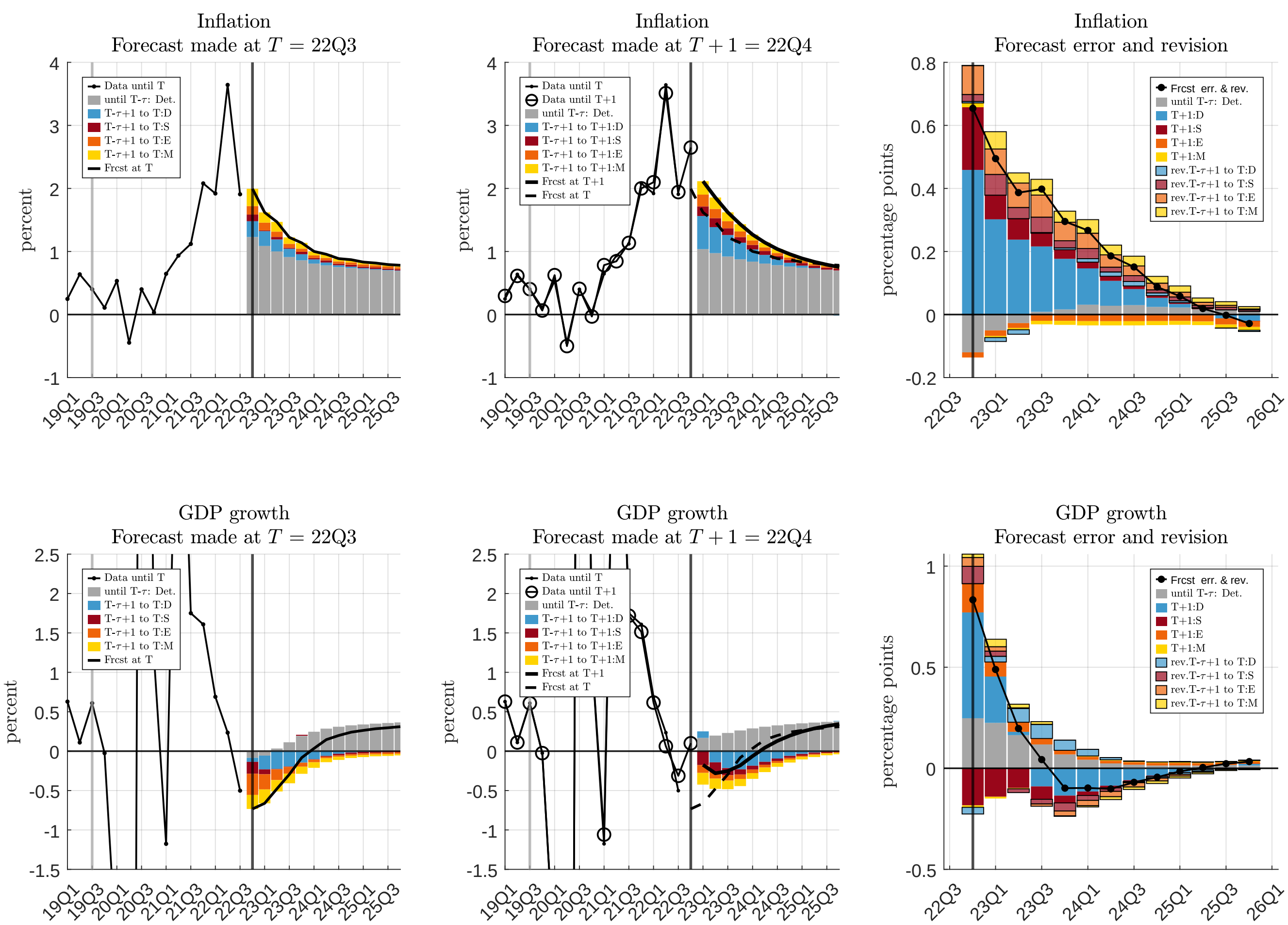
A) Shocks in 2022Q2

B) Shocks in 2022Q3



Note: The first row shows the marginal posterior distribution of shocks for 2022Q2 estimated over different vintages, with T corresponding to 2022Q2 and $T+h$ the h subsequent quarters. The bottom row shows the same for 2022Q3.

Figure B-6: Forecast analysis for 2022Q4



Note: The left-hand side panels show the forecast at time T (solid line), while the middle panels show the new forecast at time $T+1$ (solid line) along with the one produced at time T (dashed line). Both forecasts are decomposed into the role of the different components up to time T and $T+1$ respectively: demand (blue bars), supply (red bars), energy (orange bars), monetary policy shocks (yellow bars) and deterministic component (grey bars). The right-hand side panels plot the marginal difference between the forecasts, along with the contribution of each component.

Figure B-7: Series of the shocks over the last year: 2022Q2

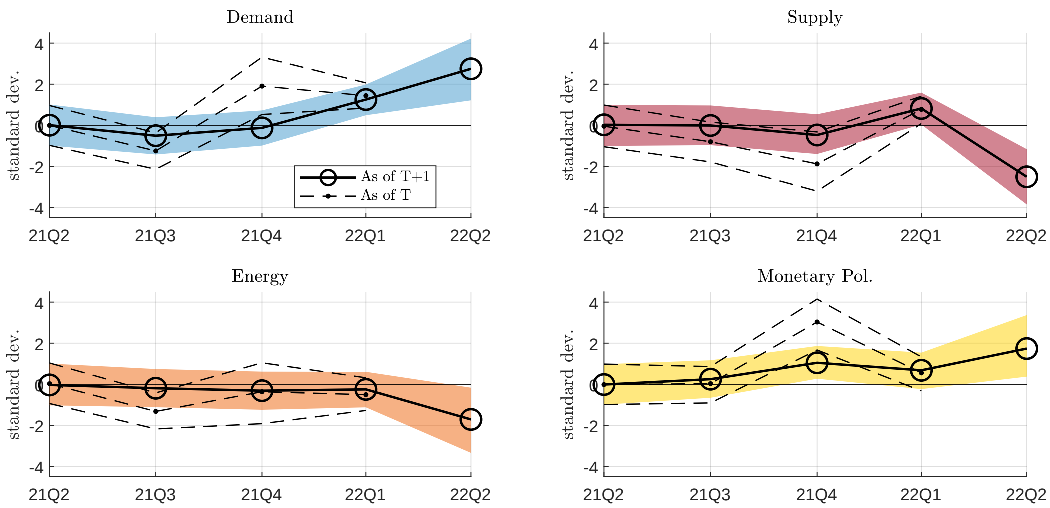


Figure B-8: Series of the shocks over the last year: 2022Q3